THE ASTROPHYSICAL JOURNAL

An International Review of Spectroscopy and Astronomical Physics

FOUNDED IN 1895 BY GEORGE E. HALE AND JAMES E. KEELER

VOLUME 83

JANUARY–JUNE 1936

THE UNIVERSITY OF CHICAGO PRESS
CHICAGO, ILLINOIS

THE CAMBRIDGE UNIVERSITY PRESS, LONDON
THE MARUZEN COMPANY LIMITED, TOKYO
THE COMMERCIAL PRESS, LIMITED, SHANGHAI

PUBLISHED JANUARY, MARCH, APRIL, MAY,
JUNE 1936

COMPOSED AND PRINTED BY THE UNIVERSITY OF CHICAGO PRESS
CHICAGO, ILLINOIS, U.S.A.

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January 1936 Vol. 83, No. 1

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The Astrophysical Journal is published by the University of Chicago at the University of Chicago Press, 5750 Ellis Avenue, Chicago, Illinois, during each month except February and August. ¶ The subscription price is $6.00 a year; the price of single copies is 75 cents. Orders for service of less than a half-year will be charged at the single-copy rate. ¶ Postage is prepaid by the publishers on all orders from the United States, Mexico, Cuba, Porto Rico, Panama Canal Zone, Republic of Panama, Dominican Republic, Canary Islands, El Salvador, Argentina, Bolivia, Brazil, Colombia, Chile, Costa Rica, Ecuador, Guatemala, Honduras, Nicaragua, Peru, Hayti, Uruguay, Paraguay, Hawaiian Islands, Philippine Islands, Guam, Samoan Islands, Balearic Islands, Spain, and Venezuela. ¶ Postage is charged extra as follows: for Canada and Newfoundland, 30 cents on annual subscriptions (total $6.30); on single copies, 3 cents (total 78 cents); for all other countries in the Postal Union, 80 cents on annual subscriptions (total $6.80), on single copies, 8 cents (total 83 cents). ¶ Patrons are requested to make all remittances payable to The University of Chicago Press, in postal or express money orders or bank drafts.

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For the British Empire, except North America, India, and Australasia: The Cambridge University Press, Fetter Lane, London, E.C. 4. Prices of yearly subscriptions and of single copies may be had on application.

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For China: The Commercial Press, Ltd., 211 Honan Road, Shanghai. Yearly subscriptions, $6.00; single copies, 75 cents, or their equivalents in Chinese money. Postage extra, on yearly subscriptions 80 cents, on single copies 8 cents.

Claims for missing numbers should be made within the month following the regular month of publication. The publishers expect to supply missing numbers free only when losses have been sustained in transit, and when the reserve stock will permit.

Business correspondence should be addressed to The University of Chicago Press, Chicago, Illinois.

Communications for the editors and manuscripts should be addressed to: Otto Struve, Editor of THE ASTROPHYSICAL JOURNAL, Yerkes Observatory, Williams Bay, Wisconsin.

The cable address is "Observatory, Williamsbay, Wisconsin."

The articles in this journal are indexed in the *International Index to Periodicals*, New York, N.Y.

Applications for permission to quote from this journal should be addressed to The University of Chicago Press, and will be freely granted.

Entered as second-class matter, January 17, 1895, at the Post-Office, Chicago, Ill., under the act of March 3, 1879.
Acceptance for mailing at special rate of postage provided for in Section 1103, Act of October 3, 1917, authorized on July 15, 1918.

PRINTED IN THE U.S.A.

EDWIN BRANT FROST, 1866–1935

THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND
ASTRONOMICAL PHYSICS

VOLUME 83 JANUARY 1936 NUMBER 1

EDWIN BRANT FROST
1866–1935

PHILIP FOX

Professor Edwin David Sanborn, of Dartmouth College, kept a notebook in which he recorded comments and opinions on his students. Of Charles Augustus Young, member of the class of 1853, he wrote: "Uncommonly quick, learning everything with great facility, excitable and playful in recitation but always prepared better than anyone else, in character faultless, in piety commendable." The youth of whom Professor Sanborn wrote this characterization became the leader of a school of astronomy, first at Dartmouth and later at Princeton. Of this school from Dartmouth we find Lewis Boss, John Robie Eastman, Henry Martyn Paul, Edwin Brant Frost. Of these, Professor Frost carried on the tradition at Dartmouth with the later generation which included such men as John Merrill Poor and Walter Sidney Adams. Dr. Frost did not have his undergraduate work with Young, but as a child he was daily in and out of Young's home with the son Fred Young his constant companion; and Professor Young more than any other shaped his career.

Professor Frost was born at Brattleboro, Vermont, July 14, 1866, of the eighth generation of his family in America, a descendant of Edmund Frost, who landed in Boston October 2, 1635, and the son of the physician Carlton Pennington Frost and Eliza Ann DuBois. He was of strictly New England stock and a New Englander to the

core. Figuratively, at least, he practiced in his life the New England motto which he often quoted, "Eat it up, wear it out, make it do." The pronunciation of such words as "coat" and "road" served as shibboleths which revealed him. The Connecticut River, Mount Ascutney, Moosilauke of the Franconias, Monadnock, Wautastiket, Chocorua, Kearsarge, the White Mountains, were of the landscape which framed his youthful vision.

It was natural that Frost should attend Dartmouth College, for his whole youth was spent in Hanover, in and about the College. His father and two uncles, Edwin and Henry, had graduated there before him; and his father had left his practice in Brattleboro to become professor of medicine at his Alma Mater in 1871.

In his autobiography, *An Astronomer's Life*, published in 1933, Frost gives a full and very entertaining account of his boyhood and young manhood. We read of his early days, of home life in a scholarly atmosphere with brilliant conversation of distinguished visitors, of music, of his boyhood friends Dean C. Worcester, Allen Hazen, Sam Bartlett. As one reads of his deep interest in trees, in animal life, especially birds, games and quests which would seem to be the awakening of a great naturalist, one must be surprised that he turned to astronomy. But here we read that Young played the decisive part in the choice of field of work, even as later Frost was to turn Adams from the classics to a brilliant career in astronomy. But Frost's choice was clinched when the new star in the Andromeda Nebula appeared and was used as the subject of his Senior oration. Frost graduated well up in his class with honors in physics, his brother Gilman, who specialized in mathematics—later to turn to medicine and become head of the medical school—leading him by a narrow margin to be class salutatorian. Perhaps Frost's greatest pleasure in college was gained in courses in calculus and analytical mechanics under the many-sided scholar, literateur, and diplomat, Professor Arthur S. Hardy.

After graduation Frost remained at Dartmouth for graduate work, serving as an assistant in the department of chemistry. He interrupted this course at the end of the first term to teach school for twelve weeks at Hancock, New Hampshire. Thereafter he went to Princeton to study with Young. In the fall of 1887 he returned to Dartmouth as instructor in physics.

PLATE II

Taber, Bolza, Moore, Maschke, White
Tyler, Story, Klein, Hough, Clark, Holgate, ?
Hale, Frost, Snow, Dowling, Blake, Douglass, Waldo, ... Keppel, ?, Updegraff, Cunningham, Shaw, Van Vleck
Brashear, Keeler, Reed, Leuschner, W. H. Pickering, Webster, W. B. Smith, Weiss, Eddy, See, Todd, Wolf, Burnham, Oliver, Rowland
?, Collins, Howe, Loud, Study, McMahon, McNeill

I suppose everyone remembers vividly the first general meeting of his scientific craft which he attended. I well recall mine in 1902 at Washington, with Hale, Pickering, Tittman, Langley, Abbot, Nichols, Comstock, Doolittle, and Newcomb among those present. I hear, as though it were yesterday, Father Brennan urging us to come to St. Louis for the next meeting, assuring us for a third, fourth, fifth time that the "good people of St. Louis will turn out and make yez all cardially wilcome," and wonder still at the imperturbability of the leonine Newcomb, presiding. Frost tells of his first meeting at Toronto, mentioning H. A. Newton and Simon Newcomb, and speaking of his meeting W. W. Campbell and the beginning of their friendship through careers which marched in similar paths.

Frost had two years abroad, spent mostly in Germany, shifting wisely from Strasbourg to Potsdam to work with Vogel and Scheiner. At Strasbourg he heard lectures by Kohlrausch, Wiener, Wislicenus, and Becker. At Potsdam the new work in astrophysics was unfolding. The appearance of Nova Aurigae gave a great impetus to Frost's study. On him fell the task of adjusting a makeshift spectrograph to photograph its spectrum, an instrument with which he photographed also spectra of stars which had bright lines, observations which defined the direction of his future career.

Half of the advantage of foreign study lies in the stimulus of acquaintanceships which are made. Frost's engaging, informal personality may have puzzled the more formal professors and fellow-students but won their friendship. Turner, Huggins, Wolf, Ristenpart, Riem, Kobold, Förster, Paetsch, Schwarzschild, Lebedew, Belopolsky, Halm, Wilsing, Scheiner, and Vogel were his lifelong friends; and whom among them would we not also gladly have hailed as friend?

Frost returned to Hanover in 1892, feeling the loss of the opportunities for research but launched into the translation of Scheiner's *Spectralanalyse der Gestirne*. Also, with the co-operation of Professor Pickering, who furnished the plates, he began the study of β Lyrae, an investigation never finished but never relinquished.

In 1893 Professor Frost attended the Columbian Exposition and its scientific congresses. An old print, Plate II, shows the group of mathematicians, physicists, and astronomers who participated. The mathematicians Study and Klein and the astronomers Max Wolf

and Weiss were the foreigners present. In the upper row left, standing together, are the three elements, Hale, Frost, and Snow, whose close association through many years was the source of many drolleries. At Chicago, also, Dr. Frost saw in the Manufacturers' Building the mounting of the great Yerkes refractor, the focus of interest even in that temple of marvels, the telescope which he was later to use for the major part of his career.

Professor George E. Hale, through whose great vision and enthusiasm the Yerkes Observatory came into being, organized its program on a very broad plan, including research in the newer fields of astronomy. In 1898 Frost was invited to join the staff as professor of astrophysics. This appointment he accepted, effective July first, and there he remained, succeeding to the directorship in 1905, when Hale organized the Mount Wilson Observatory. Though Frost took up his work at Williams Bay in 1898, each season until 1902 he spent part time in Hanover, teaching astronomy.

Dr. Frost's scientific contributions were fourfold—teacher, investigator, editor, lecturer. As a teacher I can see him sitting at his desk before his Dartmouth students with his legs curled around and back of the front legs of his chair, direct, clear, stripping away nonessentials, revealing enthusiasm for the subject in which he was then gaining recognition as a leader, knowing his students as friends and so regarded by them. As I review the characteristics of the many teachers whom I have known he stands easily among the first, a great teacher.

As an investigator, the major part of his work was in the field of stellar spectroscopy, especially in determinations of radial velocity and the problems of spectroscopic binaries. But in his early career at the Shattuck Observatory he conducted systematic solar observations, observations of comets with a ring micrometer, kept the meteorological records, and made there some of the earliest X-ray experiments conducted in America. The interest in solar problems, inherited from Young, found expression in his examination at Potsdam of the radiation from various parts of the solar disk and in the eclipse expeditions to Wadesboro, Green River, and Santa Catalina. The first of these expeditions, conducted by Dr. Hale, was successful. Frost's observations were published[1] in a valuable paper, "Spectro-

[1] *Ap. J.*, **12**, 307, 1900.

scopic Results obtained at the Solar Eclipse of May 28, 1900." In his principal field, stellar spectroscopy, he published many notes on individual stars, including the announcement of many spectroscopic binaries, and four major papers. The first of the four, "Radial Velocities of Twenty Stars Having Spectra of the Orion Type,"[2] was published with Dr. Walter S. Adams, who co-operated with him in much of the early work with the Bruce spectrograph. In this paper is found the first recognition of systematically different velocities for stars of different spectral classes. The second was an outgrowth of Dr. Frost's suggestion that various observers of radial velocities prepare a list of standard velocity stars to test their instrumental equipment and methods, for accuracy, constancy, and systematic errors—a suggestion which met with cordial response. The paper by Frost and Adams[3] contained results for the adopted stars. The results by the two measurers, plate for plate, and for the various plates of the same star, were in excellent agreement. The third, "Radial Velocities of 368 Helium Stars," was published[4] under the joint authorship of Frost, Barrett, and Struve. The fourth, "Radial Velocities of 500 Stars of Spectral Class A,"[5] was of the same joint authorship. These papers present a vast amount of observational material of high value.

As an editor we must think first of his translation and revision of Scheiner's *Spectralanalyse der Gestirne*, which he published in 1894 under the title *Astronomical Spectroscopy*. One needs only glance through its pages to see on what a sure foundation this field of research was established and to see what tremendous advances have been made since its publication. On the first volume of the *Astrophysical Journal* in 1895, Frost's name appears as assistant editor. In 1902 he became editor. For more than thirty years he served in that capacity. Through eighty-one volumes his name appeared on the editorial staff, and for most of them the editorial task was principally his. To these duties he gave meticulous attention, perhaps too much to the chore of proofreading. The scarcity of typographical errors and the detection and elimination of many an author's error, thus sparing embarrassment, is perhaps justification enough. In its pages are found many of his scientific contributions and numerous

[2] *Pub. Yerkes Obs.*, **2**, 145–250, 1904.

[3] *Ap. J.*, **18**, 237, 1903.

[4] *Ap. J.*, **64**, 1, 1926.

[5] *Pub. Yerkes Obs.*, **7**, 1, 1929.

reviews. I believe he never wrote a review without reading the paper or book with care. Mention must be made of his editorship of the extensive solar observations of Dr. C. H. F. Peters published by the Carnegie Institution, *Heliographic Positions of Sun-Spots Observed at Hamilton College from 1860 to 1870*, and of his co-operation with Comstock, Barnard, and Pickering in compiling the report of the Comet Committee of the American Astronomical Society.[6]

On the night of December 15, 1915, Dr. Frost suffered an impairment to his vision which permanently put an end to active observing. The impairment resulted from a detached retina in the right eye. Within a year the beginning of a cataract in the left eye threatened complete blindness. He writes: "Then in 1921, 'adding insult to injury' a hemorrhage occurred in this eye which made reading impossible." We find him rejoicing later in the clearing of the hemorrhage and the restoration of 3 or 4 per cent vision. This, however, lessened with the progressive development of the cataract. This cruel malady —and what could be harsher than blindness to an astronomer?—deafness to a musician perhaps—brought out such magnificent traits of character and aroused in those who saw him go about his ways in cheerful mien, such quick sympathy and profound admiration that the influence of his dark years upon his fellow-men was perhaps even greater than those devoted actively to research.

Failing and lost vision did not terminate his scientific career. He continued as director of the Yerkes Observatory and active editor of the *Astrophysical Journal* until 1932, when, having reached the normal age of retirement, he became Director Emeritus. He has written: "With a good memory and an adequate imagination, one can see much without the use of eyes." Articles were read to him, progress of work under way reported, new lines of research explained. In writing of his early education, Dr. Frost comments on the severe discipline of carrying through Colburn's *Mental Arithmetic:* "It gave us the habit of thinking quickly and clearly without depending on pencil and paper . . . . we thus early acquired the habit of forming a mental picture of the whole problem straight through to its solution." This early training stood him nobly in hand when sight was gone. Exceedingly sensitive and accurate hearing, trained in music, acute

[6] *Pub. A.A.S.*, **2**, 177–218, 1915.

in the detection of scarcely audible bird calls and his whistled reproductions of those calls, able to recognize the engines of the Northwestern and St. Paul railroads and to match infallibly the engine number with the sounding whistle, schooled by the acquisition of foreign languages, the German and French which he loved to speak; a delicate sense of touch interpreting the brush of air currents as proximity of a wall or building; and a keen sense of smell, accurately distinguishing various roses and flowers, led him to find strength, joy, and entertainment in new-found powers.

He listened to the crickets chirp and derived the formula associating the rate with temperature, $y=ax+b$, where y is the temperature, x is the number of chirps per minute, b the temperature at which the chirping stops, 42° F., and a the constant number 13. This is partly the result of his early interest in natural history and partly to be credited to the quickened senses barred in one avenue and seeking an outlet by other means.

During this later period he lectured widely, instructively, and entertainingly. The solid science presented with dignity but often enlivened with whimsical humor and ready wit—quaint, homely, informal address soaring at times to lofty expression—carried a unique impressiveness. Blindness passed from his eyes in making his audience see and understand. In his lecture tours and on the platform his wife was his constant companion and aid.

Perhaps the most thrilling occasion in which he participated was on the opening night of the Century of Progress Exposition. He had suggested the appropriateness of adding "a celestial touch to this ceremony of illuminating the Century of Progress by employing the light of the great star Arcturus" to throw the switch to set the exposition lights ablaze. From the rostrum high above the massed throng he spoke impressively. After the ceremony I drove with him and Mrs. Frost through the avenues, the only car on the grounds, explaining as best one might to one who could not see. He was everywhere recognized and acclaimed.

Dr. Frost was a member of many learned societies and academies. Among them the National Academy of Sciences, the American Academy of Arts and Sciences, the American Philosophical Society. He was a foreign associate of the Royal Astronomical Society. His Al-

ma Mater gave him recognition by awarding, in 1911, the honorary degree of Doctor of Science and the University of Cambridge its doctorate in 1912.

In summing up Dr. Frost's characteristics, one thinks instinctively of Richard Hovey's "Men of Dartmouth." Dick Hovey, who lived in the Frost home, has expressed the ruggedness of early Dartmouth where the spirit of Webster and Choate still stirred:

"They have the still North in their hearts
The hill winds in their veins
And the granite of New Hampshire
In their muscles and their brains."

The serenity, the dignity, the cleanliness of the hills—save for the flashes of wit one might almost say their austerity—were his birthright. Dr. Frost was a lover of outdoors and all its creatures. "Let the unfortunate city dweller rightly proclaim the beauty of the city's skyline at dusk, or praise the architecture of its buildings; let him rejoice in the opportunities which the city has to offer. As for me and mine, we like the open spaces with their sense of freedom, their beauty, and their calm." In writing "and mine" Professor Frost was thinking of his wife and three children, Katharine, Frederick, and Benjamin. He was married in 1896 to Miss Mary E. Hazard, of Boston, who shared his life in all aspects fully. Their family was one of mutual understanding, appreciation, respect, and the home one of most cordial hospitality.

He was a lover of birds, wise in their ways; a lover of flowers, successful in their culture, as his garden full of roses attested. He shared with his wife a love of music, with wide acquaintance of its literature, singing with true, clear, and sympathetic tenor voice. He was a devotee of sport, graduating from his boyhood tree climbing to mountain climbing, playing occasional golf and swimming in summer, but loving best his New England winter sports, coasting and skating.

Under restriction and its hours of meditation and reflection he found a philosophy which calls for an orderly universe, a purposeful creation. Under affliction he went his way "sustained and soothed by an unfaltering trust." Sightless eyes may not serve well to guide

one's steps, but into what graver pitfalls a sightless soul may stray. In him there was charity so broad, insight so deep, courage so superb, confidence so serene, reliance so adequate, that we find full exemplification of the verse:

> "But often faltering feet
> Come surest to the goal,
> And they who walk in darkness meet
> The sunrise of the soul."

ADLER PLANETARIUM AND ASTRONOMICAL MUSEUM
CHICAGO, ILLINOIS
November 11, 1935

THE APPARENT RADIAL VELOCITIES OF 100 EXTRA-GALACTIC NEBULAE*

M. L. HUMASON

ABSTRACT

New velocities of 100 nebulae are given in Table I. With the exception of 6, they are all velocities of recession ranging from +50 km/sec, for a member of the Virgo Cluster, to +42,000 km/sec for a member of the Ursa Major Cluster No. 2.

In the Virgo Cluster 25 new velocities have been obtained, 9 of them by Sinclair Smith. These, with velocities previously known, show an average range of 500 km/sec around a mean of +1200 km/sec. Magnitudes range from 10.0 to 15.0. The mean velocity of the fainter members is approximately the same as that of the brighter.

Late-type spirals (Sc) are decidedly bluer than E, Sa, or Sb nebulae. The mean spectral type for these groups is: E0–7, G3.6; Sa, G3.4; Sb, G1.6; Sc, F8.8.

Velocities of the order of +40,000 km/sec have been observed in the Boötes Cluster and in the Ursa Major Cluster No. 2. These values are consistent with estimates of the distance by Hubble and Baade (about 70 million parsecs for both clusters) and indicate that the velocity-distance relation is still sensibly linear to this distance.

Since the first formulation of the velocity-distance relation for extra-galactic nebulae by Hubble, the program of nebular velocities at the Mount Wilson Observatory has been planned (*a*) to extend the range in distance to the observable limit of the 100-inch reflector by determining velocities of the brightest members in faint clusters of nebulae, (*b*) to investigate the dispersion in velocity of bright and faint nebulae in selected clusters at different distances, and (*c*) to obtain a large sample collection of velocities of both bright and faint isolated nebulae.

The first results of the investigation, the velocities of 46 nebulae, were published in 1931.[1] The present investigation includes velocities from 6 clusters, 5 groups, and 56 isolated objects, a total of 100 new velocities, which are listed in Table I. Nine of the velocities from the Virgo Cluster were obtained with the 60-inch reflector and measured by Sinclair Smith. In addition, Table II contains the velocities and spectral types of five nebulae whose velocities had previously been announced by Slipher and others. They are included for purposes of comparison with the values obtained by other observers with different instruments.

* *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 531.

[1] *Mt. W. Contr.*, No. 426; *Ap. J.*, **74**, 35, 1931.

TABLE I

APPARENT VELOCITIES OF 100 EXTRA-GALACTIC NEBULAE

NGC	1950 R.A.	1950 Dec.	Pg *m*	Type	Velocity	Sp.	Remarks
7814	$0^h\ 0^m7$	$+15°51'$	12.4	Sa	+ 1000::	G3	
68	0 15.8	+29 48	......	E	+ 5700	G3	
69	0 15.8	+29 46	......	Sa	+ 6700	G0	
71	0 15.8	+29 47	......	E	+ 6600	G3	Group
72	0 15.9	+29 46	......	SBb	+ 7000	G7	
Anon 1	0 16.0	+29 46	......	Sa	+ 6800	G3	
80	0 18.6	+22 5	......	E	+ 5600	G5	Group
83	0 18.8	+22 9	......	E	+ 6500	G3	
157	0 32.3	− 8 40	11.1	Sc	+ 1800	G4	
160	0 33.4	+23 41	......	Sa	+ 2600::	G8	
* 247	0 44.6	−21 1	10.7	Sc	− 15	Pd	
* 253	0 45.1	−25 34	7.0	Sc	− 50	Pd	
* 379	1 4.5	+32 15	......	Sa	+ 5500	G6	Group
628	1 34.0	+15 32	11.2	Sc	+ 600:	G2	
681	1 46.7	−10 40	12.9	S	+ 2100:	G5	
720	1 50.6	−13 59	11.7	E5	+ 1800:	G4	
* 772	1 56.6	+18 46	12.0	Sb	+ 2200	G4	
* 1084	2 43.5	− 7 47	11.2	Sc	+ 1500	F5	
1087	2 43.9	− 0 42	11.2	Sc	+ 1850	F0	
1332	3 24.1	−21 31	11.4	Sa	+ 1400	G2	
1400	3 37.2	−18 51	12.5	E1	+ 500	G5	
1407	3 37.9	−18 44	11.5	E0	+ 2000:	G8	
* IC 342	3 41.9	+67 57	14.0	Sc	− 25	F8e	
Anon 2	4 38.1	+ 4 10	16.0	Sa	+ 4600	G7	
Anon 3	7 4.4	+35 9	17.0	E?	+24000::	G2	Gem Cl.
Anon 4	7 5.0	+35 4	16.8	E	+23000	G3	
2403	7 32.0	+65 43	10.2	Sc	+ 125	F8	
* 2655	8 49.4	+78 25	11.6	Sa	+ 1350	G2e	
2775	9 7.7	+ 7 15	11.5	Sa	+ 1100	G3	
2768	9 7.8	+60 16	12.0	E6	+ 1400	G3	
2787	9 14.9	+69 25	12.1	SBa	+ 700	G5	
2903	9 29.3	+21 44	10.3	Sc	+ 350	F0	
2985	9 46.0	+72 31	11.8	Sb	+ 1050:	G3	
3147	10 12.8	+73 39	11.9	Sc	+ 2600	G5	
* 3344	10 40.7	+25 11	11.9	SBc	+ 550	G0e	
3351	10 41.3	+11 58	11.5	SBb	+ 675	F5	
* 3377	10 45.1	+14 15	11.6	E5	+ 650	G2e	
3384	10 45.7	+12 54	11.3	SBa	+ 850	G0	
3412	10 48.3	+13 41	11.6	SBa	+ 950	G0	
3414	10 48.6	+28 15	12.2	SBb	+ 1450	G7	

TABLE I—*Continued*

NGC	1950 R.A.	1950 Dec.	Pg *m*	Type	Velocity	Sp.	Remarks
Anon 5	$10^h55^m.4$	+57° 3′	17.5	E	+19000::	G0	
Anon 6	10 55.7	+57 1	17.9	E	+42000::	G2	UMa Cl. No. 2
3486	10 57.8	+29 15	11.4	Sc	+ 1250:	G3	
3726	11 30.7	+47 19	11.7	Sc	+ 1150::	G0	
* Anon 7	11 44.5	+56 1	15.9	E	+15400	G5	UMa Cl. No. 1
4216	12 13.4	+13 25	11.3	Sb	+ 50	G3	
4261	12 16.8	+ 6 6	11.7	E2	+ 2300	G5	
4267	12 17.2	+13 3	12.6	E	+ 1200	G5	
4281	12 17.8	+ 5 40	12.2	Sa	+ 2900	G3	
4303	12 19.4	+ 4 45	10.4	SBc	+ 1900	G0	
4321	12 20.4	+16 6	10.8	Sc	+ 1650	F2	
4365	12 22.0	+ 7 36	11.0	E3	+ 1200	G3	
4394	12 23.4	+18 29	12.2	SBb	+ 850	G4	
† 4421	12 24.5	+15 44	......	Sa	+ 1300:	G5	
4429	12 24.9	+11 23	11.7	Sa	+ 1300	G5	
4435	12 25.2	+13 21	11.8	E6	+ 950	G3	
4442	12 25.6	+10 5	11.4	SBa	+ 650	G5	
4450	12 25.9	+17 21	11.4	Sb	+ 2100	G3	Vir Cl.
† 4458	12 26.4	+13 31	......	E0	+ 300:	G5	
† 4464	12 26.8	+ 8 26	......	E1	+ 850::	G5	
† 4467	12 27.0	+ 8 16	......	E2	+ 1600::	G5	
† 4473	12 27.3	+13 42	11.7	E5	+ 2300	G8	
4477	12 27.6	+13 55	11.8	SBa	+ 1300	G5	
† 4478	12 27.8	+12 36	12.5	E2	+ 1550	G5	
4479	12 27.8	+13 51	......	SBa	+ 1300::	G3	
† Anon 8	12 28.0	+12 46	......	E0	+ 1400::	G3	
† 4492	12 28.4	+ 8 21	......	Sa	+ 1600	G3	
† 4551	12 33.1	+12 33	......	E4	+ 1000::	G5	
* 4569	12 34.3	+13 26	11.2	Sb	− 200:	F0	
4621	12 39.5	+11 55	11.4	E5	+ 500	G6	
4725	12 48.1	+25 46	10.8	Sb	+ 1100	G3	
* 5253	13 37.1	−31 24	10.8	Ir	+ 425	Pd	
5322	13 47.6	+60 26	11.6	E	+ 1900	G5	
5566	14 17.8	+ 4 11	11.9	SBb	+ 1550	G5	
Anon 9	14 30.6	+31 46	17.8	E	+39000::	G7	Boö Cl.
5746	14 42.3	+ 2 10	11.8	Sb	+ 1850:	G5	
5846	15 4.0	+ 1 48	11.6	E0	+ 1700	G5	
5907	15 14.6	+56 31	11.8	Sc	+ 400::	G2	
Anon 10	15 20.3	+27 53	16.7	E2	+21000::	G2	CrB Cl.
5982	15 37.6	+59 32	12.5	E3	+ 2900	G4	

TABLE I—*Continued*

NGC	1950 R.A.	1950 Dec.	Pg *m*	Type	Velocity	Sp.	Remarks
5985....	$15^h38^m.6$	$+59°30'$	12.2	Sb	+ 2600::	G4	
6181....	16 30.1	+19 56	12.6	Sc	+ 2500:	G2	
* 6207....	16 41.3	+36 56	12.3	Sb	− 250:	F2	
6340....	17 11.1	+72 22	12.8	E	+ 2100::	G8	
6384....	17 29.9	+ 7 6	12.7	SBb	+ 1450:	G8	
6574....	18 9.5	+14 58	12.7	Sc	+ 2550::	G0	
6946....	20 33.9	+59 58	11.1	Sc	− 150	Pd	
Anon 11....	20 58.5	+16 7	......	E	+ 9200::	G3	Group
Anon 12....	20 58.8	+15 56	......	Sbc	+12400::	G3	Group
Anon 13....	20 59.6	+15 56	......	E	+11100::	G2	Group
7177....	21 58.3	+17 29	12.1	Sbc	+ 1200:	G2	
7171....	21 58.3	−13 31	12.8	S	+ 2600::	G3	
7318....	22 33.7	+33 42	......	E	+ 6700::	F8	Group
7319....	22 33.8	+33 42	......	SBc	+ 6100::	F8	Group
7332....	22 35.0	+23 32	12.6	S	+ 1300:	G4	
7343....	22 36.4	+33 48	14.1	SBb	+ 1200::	F8	
7448....	22 57.6	+15 43	11.8	Sc	+ 2300::	G3	
7457....	22 58.6	+29 53	12.3	Sa	+ 500:	G2	
7727....	23 37.3	−12 34	12.0	Sa	+ 1800	G5	
* 7741....	23 41.4	+25 48	12.6	SBb	+ 300:	Foe	

NOTES TO TABLE I

The apparent photographic magnitudes and nebular types are from *Harvard Annals*, **88,** No. 2, or from Mount Wilson estimates made by Hubble. Values of the velocity less than 2000 km/sec are given to the nearest 50 km/sec; those larger, to the nearest 100 km/sec. The estimated uncertainty of the velocities is of the order of 50 km/sec. Values marked with one colon indicate the uncertainty may be as large as 100 km/sec; those marked with two colons indicate the uncertainty may be as large as 200 km/sec. The spectral types are Mount Wilson estimates.

A dagger in the first column indicates that the spectrogram was obtained and the velocity measured by Sinclair Smith. An asterisk in this column refers to the following notes:

NGC 247. Velocity from emission patch Np nucleus.

NGC 253. Velocity from emission patch N of nucleus.

NGC 379. A member of the Pisces group.

NGC 772. In *Proc. Nat. Acad.*, **20,** 265, 1934, this object was by mistake listed as NGC 722.

NGC 1084. λ 3727 probably bright.

IC 342. λ 3727 bright.

NGC 2655. λ 3727 bright.

NGC 3344. λ 3727 bright.

NGC 3377. λ 3727 bright.

Anon 7. Corrected velocity for the brightest nebula in Ursa Major Cl. No. 1.
NGC 4569. Velocity may be that of a star projected on the nucleus.
NGC 5253. Possibly not extra-galactic.
NGC 6207. Velocity may be that of a star projected on the nucleus.
NGC 7741. λ 3727 bright.

All the observations have been made with the two nebular spectrographs at the Cassegrain focus of the 100-inch and 60-inch reflectors. When used with two prisms, the dispersion in angstroms per

TABLE II

NGC	M.W.		SLIPHER	M.W. SANFORD	LICK
	Vel.	Spectrum			
278*	+700	A0	+650	....	....
404	20	G0	− 25	....	....
3379†	850	G4	+780	+845	....
4151§	953	....	+980	....	+940
5866	+725	G2	+650	....	....

* Slipher classifies the spectrum as B.
† Velocity from two spectrograms taken by F. G. Pease.
§ Velocity measured from emission lines on large-scale plates, estimated uncertainty about 5 km/sec.

millimeter at λ 4350 is 440 for spectrograph VI and 510 for spectrograph VI A. Both spectrographs are equipped with Rayton cameras, and the large majority of the observations were made with the two-prism dispersion.

CLUSTER NEBULAE

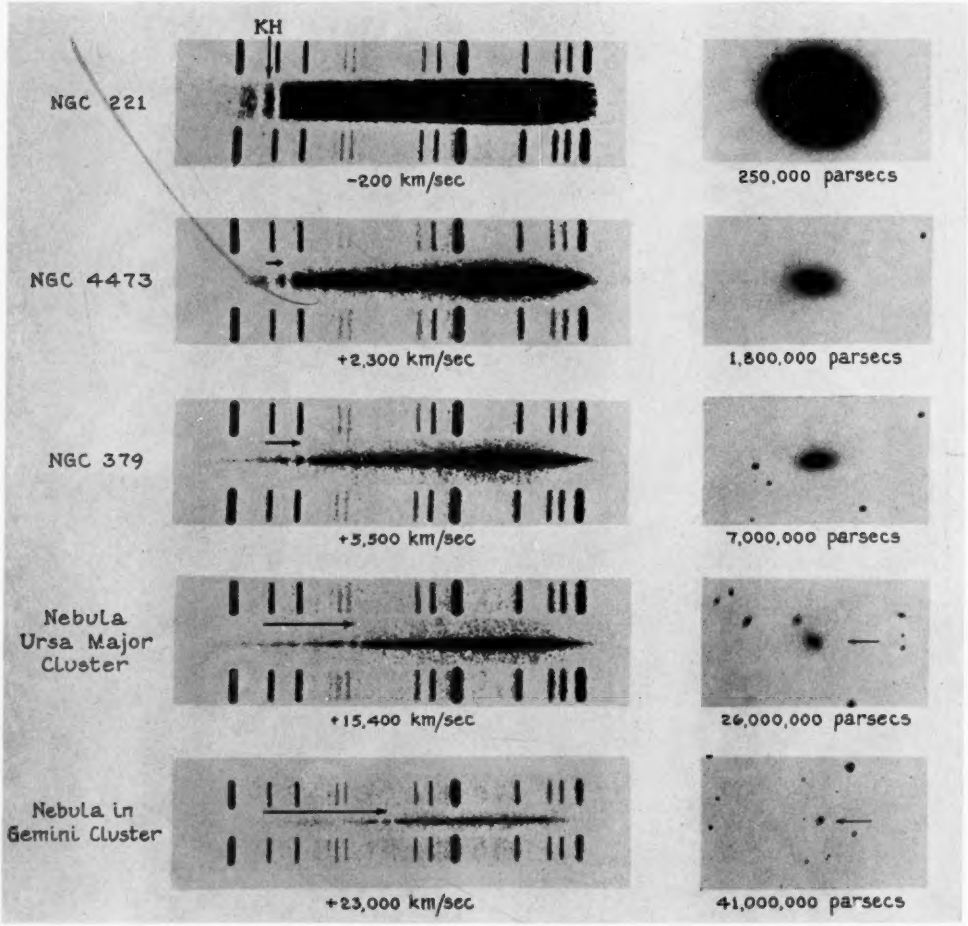
Virgo Cluster.—Velocities of 25 nebulae have been obtained. These, together with two values previously published at Mount Wilson and five by V. M. Slipher from the Lowell Observatory, show an average range of 500 km/sec around a mean of +1200 km/sec. The range in pg m is from 10.0 to 15.0, and there is no change in the size of the velocities with decreasing brightness. A more detailed study and analysis of the velocities, made by Sinclair Smith, may be found in the following *Contribution.*[2]

Ursa Major Cluster No. 1.[3]—In *Mount Wilson Contribution* No.

[2] *Mt. W. Contr.*, No. 532; *Ap. J.* **83**, 23, 1936.

[3] The Virgo Cluster and Ursa Major No. 1 are described by Hubble and Humason in *Mt. W. Contr.*, No. 427; *Ap. J.*, **74**, 43, 1931.

PLATE III



RED-SHIFTS IN THE SPECTRA OF EXTRA-GALACTIC NEBULAE

Arrows above the spectra (enlarged twenty times from the original negatives) point to the H and K lines of calcium and show the amounts these lines are displaced toward the red. The comparison spectra are of helium.

The direct photographs (on the same scale and with approximately the same exposure times) illustrate the decrease in size and brightness with increasing velocity or red-shift.

NGC 4473 is a member of the Virgo Cluster; NGC 379 of a group in Pisces.

426 the velocity of the brightest member, Baade No. 24, pg $m = 15.9$, is given as $+11,700$ km/sec. This velocity was measured on a one-prism spectrogram of very poor quality obtained during a short period when film was being used in place of glass plates. The use of film was finally abandoned on account of its tendency to crawl in the plateholder during long exposures, and this object has recently been reobserved with the two-prism dispersion. The recent plate shows definitely that the velocity is $+15,400$ km/sec (see Pl. III). The new value is in good agreement with Baade's recent determination of the distance of the cluster—26 million parsecs, based on photometric data obtained with the 100-inch reflector.

Gemini Cluster No. 1 (R.A. 7^h3^m7, Dec. $+35°6'$, 1930; gal. long. 150°, lat. $+20°$).—According to Hubble the cluster is not typical, having only about 200 members scattered over an area 30′ in diameter. The magnitude of the brightest nebula is 16.8, and the distance of the cluster is uncertainly estimated as of the order of 35 million parsecs. The velocities of the two brightest members have been obtained with the one-prism dispersion, and recently the value for the brighter of the two was confirmed by measures of a two-prism spectrogram.

Corona Borealis Cluster (R.A. 15^h19^m3, Dec. $+27°56'$, 1930; gal. long. 10°, lat. $+55°$).—A cluster of about 400 nebulae, covering an area approximately 30′ in diameter; discovered by Hubble, who estimates the distance as 37 million parsecs. The cluster is extended and comparable in size with the Virgo and the Coma clusters; but because of its distance, direct photographs show it as a fine example of an apparently compact cluster. A single velocity has been obtained from the brightest member, magnitude 16.7, and at least 10 others are bright enough to be observed.

Boötes Cluster (R.A. 14^h29^m8, Dec. $+32°3'$, 1930; gal. long. 17°, lat. $+66°$).—Discovered by Hubble in 1934, this cluster includes more than 150 members concentrated within an area 15′ in diameter. The magnitude of the brightest member is 17.8; the faintest members are below the limit of the 100-inch reflector. Only the brightest member has been observed; but its velocity is checked by measures from three spectrograms and is in good agreement with the distance, estimated by Hubble to be 70 million parsecs. The velocity from this

cluster represents the last well-established point on the velocity-distance diagram and indicates that the relationship is still sensibly linear out to this distance.

Ursa Major Cluster No. 2 (R.A. $10^h55^m.0$, Dec. $+57°9'$, 1930; gal. long. 115°, lat. $+55°$).—Discovered by Baade in 1931, the cluster probably includes some 200 members concentrated in an area 10′ in diameter. The magnitude of the brightest member is 17.9, and the distance is estimated as 72 million parsecs. As in the Boötes Cluster, the number of nebulae is indeterminate because the fainter ones are below the limit of the 100-inch reflector. The velocity of +42,000 km/sec from one of the brightest members is the largest so far observed, but is to be considered as somewhat uncertain until checked by one or more additional spectrograms.[4]

GROUP NEBULAE

Group near NGC 68 (R.A. $0^h14^m.8$, Dec. $+29°40'$, 1930; gal. long. 83°, gal. lat. $-32°$).—An area approximately 36′ in diameter includes 24 outstanding nebulae. Magnitudes of the brighter ones are $14.0\pm$, NGC 68 being one of the brightest and largest. Lundmark[5] estimates that some 150 objects are in the group. The 5 nebulae which have been observed show an average range of about 300 km/sec around a mean of +6600 km/sec.

Group near NGC 83 (R.A. $0^h17^m.7$, Dec. $+22°3'$, 1930; gal. long. 83°, lat. $-40°$).—Some 20 conspicuous nebulae are in this field, which covers an area 21′ in diameter. The two velocities so far obtained differ by 900 km/sec, the mean being +6000 km/sec. Magnitudes of the brighter members are of the order of 14.0.

Group near NGC 7006 (R.A. $20^h58^m.2$, Dec. $+15°55'$, 1930; gal. long. 32°, lat. $-20°$).—The position given is that of the globular star cluster NGC 7006, which is near the center of this group of about 15 nebulae. The mean velocity from three of the brightest members is +10,900 km/sec. No photometric data are available.

[4] In Table I the nebula listed as Anon 5 was first observed on the chance that it might be an outstandingly bright member of the cluster. The size of the velocity, +19,000 km/sec, and the fact that the nebula is brighter than the estimated upper limit of the cluster nebulae, indicate, however, that the object is a field nebula and not a real member.

[5] *Meddelanden från Astron. Obs. Upsala*, No. 21; *Arkiv för Matematik, Astronomi och Fysik*, **20** A, No. 13, 1927.

Group near NGC 7320 (R.A. 22^h32^m9, Dec. $+33°35'$, 1930; gal. long. 62°, gal. lat. −22°).—Five nebulae are outstandingly bright, and several fainter objects appear in the field. The velocities are from the two nebulae which form the double NGC 7318–19. No photometric data are available.

ISOLATED NEBULAE

Thirty-five velocities of isolated nebulae in Table I have already been published in the *Proceedings of the National Academy of Sciences*, **20**, 264, 1934, where, together with velocities previously known, they were used by Hubble and the writer to derive the velocity-distance relation for isolated nebulae. The frequency distribution of the residuals, which represents the luminosity function of nebulae, was also derived in that discussion and will not be revised here, since the new data would contribute little weight to the result. Moreover, the feature most essential for a significant redetermination, namely, the revision of the scale of nebular magnitudes on a definitive basis, is still in progress.

Together with values previously known, the velocities of over 100 isolated nebulae are now available whose apparent magnitudes range from about 7.5 to as faint as 17.5. With the exception of those which belong to our own local group and those which are fainter than 13.0, the corrected velocities of the remainder show no dependence on position in the sky or on nebular type. For this comparison the velocities were reduced to a given apparent magnitude (11.5) and corrected for solar motion; further, the apparent magnitudes (from the Harvard survey or from Mount Wilson estimates) were corrected in accordance with the secant law for local obscuration.

Only six nebulae in the table have negative velocities. NGC 247, 253, and IC 342 are large and relatively near objects; and the velocities of two of them, NGC 247 and IC 342, are of the same order as the estimated uncertainty of the measures, thus making the algebraic sign of no significance.

The spectra of two others, NGC 4569 (M 90), a member of the Virgo Cluster, and NGC 6207, may be those of stars projected on the nuclei. On direct photographs and at the Cassegrain focus of the 100-inch reflector the nucleus of both objects appears decidedly

stellar. In addition, absorption lines are narrower and sharper than in ordinary nebular spectra, and the spectral type is earlier. The displacements, however, are large for stellar velocities, even assuming a large estimated uncertainty in the measures. Four of the objects are classed as Sc and two as Sb.

DETERMINATIONS OF SPECTRAL TYPES

Comparisons of observed spectral types with colors made in a previous investigation[6] suggest a color excess of the order of 0.3 mag. Unpublished investigations of color indices by Hubble and by Baade, and more recently by Stebbins and Whitford, indicate a color excess of the same order for many of the objects in the present list. Further, in *Harvard Circular* 404, Whipple finds a color excess of the same order for about forty nebulae brighter than 13.0, distributed around three centers located in or near the Virgo cluster. As the velocity displacements, for the nearer nebulae at least, can account for only a small fraction of the observed color excess, the source may possibly be within the nebulae themselves or in the spectral classification. For this reason it seems desirable to give in more detail the method employed at Mount Wilson in classifying nebular spectra.

Except for the bright-line nebulae, classifications are based on comparisons with spectra of NGC 221 (M 32), and, unless otherwise noted, always apply to the nuclear region of the object classified. NGC 221 was chosen as the standard type because good-quality plates having a dispersion of 73 A per mm at λ 4350 were available. From these plates Adams, Joy, and the writer have estimated the type as G3; and from the relative intensities of the pairs of absolute-magnitude lines commonly used for stars, a value of +4.0 for the spectroscopic absolute magnitude. Absorption lines in NGC 221 are wider and shallower than in normal stellar spectra, presumably owing to a dispersion in the velocities and the spectral types of stars near the nucleus. The value dG3 is based on the strength of the metallic lines, the absorption at λ 4226, the absorption that forms the G band, and the strength of the hydrogen lines. It represents a mean value derived from a consideration of all criteria: the strength

[6] Hubble and Humason, *Mt. W. Contr.*, No. 427; *Ap. J.*, **74**, 56, 1931.

of the hydrogen lines, for instance, indicates a type somewhat earlier than G3, whereas the strength of λ 4226 and of the G band corresponds to a type somewhat later.

As an aid in making the comparisons, additional spectra of NGC 221 were obtained with different dispersions, slit-widths, and exposure-times, in order that the comparison might always be with a spectrum similar in appearance and scale. As only the strongest absorption features show in small-scale spectra, the most important criteria available for classification are the strength of the H and K lines and that of the G band. Distribution of intensity in the continuous spectrum is not considered. When seen, the hydrogen lines and λ 4226 are also given consideration. The presence or absence of these lines may not always indicate a real difference in spectral type, however, but may possibly be due to unrecognized characteristics in the nebulae themselves. Objection might therefore be made to the use of any individual nebula as a standard type on the ground that the spectrum was characteristic of that object alone. An example of the difference which may occur in the spectra of two different objects and not be noticeable on small-scale plates is furnished by large-scale spectra of NGC 221 and 224. Absorption lines in 221 are approximately twice as wide, and in 224 almost four times as wide, as the same lines in the spectrum of skylight. Further, absorption lines are stronger and better defined in spectra of type-E and Sa nebulae, than in those of later types, which may possibly be due to a smaller rotational component in early-type objects. Standardization of photographic plates and photometric measurements of the spectra may afford some help toward a more accurate classification of nebular spectra. In the meantime, however, the spectrum of NGC 221, selected as the standard type, is taken as a close representative of the mean spectral type of nebulae in general.

In Figure 1 spectral type is plotted against nebular type. The material includes 90 objects from Table I and 46 additional objects whose spectra had previously been classified and for which Hubble has recently determined the types. The figure shows that late-type spirals (Sc) are decidedly bluer than E or Sa nebulae. The mean spectral types for the groups are given in Table III.

The mean spectral type of nebulae fainter than 13.0 (isolated and cluster) is approximately the same as that of the brighter nebulae. The mean spectral type of all the nebulae in Table I is G2.5.

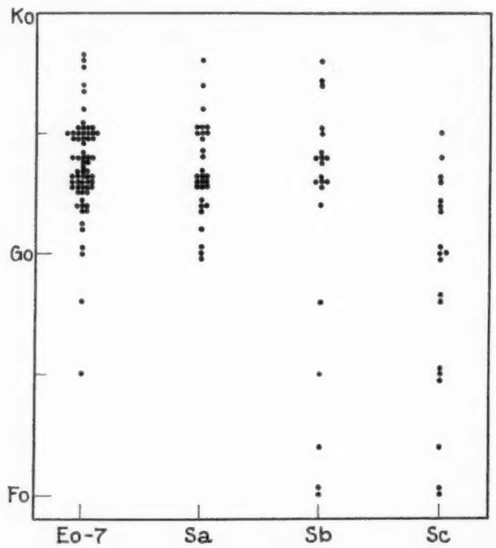


FIG. 1.—Types of nebulae plotted against spectral type. The data include 90 objects from Table I and 46 from previous investigations.

In several isolated objects the nebular emission line at λ 3727 [*O* II] has been observed. No other lines are bright, although weak emission may be present at other places in the spectrum and not recognized as such on account of the small scale or the overexposure occurring in other regions when λ 3727 is well shown. Only a relatively few spectra are well enough exposed to show this region, and it is probable that the number of nebulae in which λ 3727 occurs is considerably greater than present observations would indicate. In most cases the line is observed only in the region of the nucleus, although in one instance, NGC 7741, it extends away from the nucleus and is of uniform intensity over that part of the system covered by the slit, 6″ each side of the nucleus. The aluminizing of the Mount Wilson reflectors will help considerably in future observations of this region of the spectrum.

The nebulae observed in the two most distant clusters are too faint to be seen visually at the Cassegrain focus of the 100-inch reflector. Consequently they were placed on the slit of the spectrograph by setting off the distance from the nearest star bright enough to be seen on the slit by movement of the guiding eyepiece. For this purpose a screw adjustment and scales, which can be read to 0.05 mm. were designed for the eyepiece. The required displacement of the guid-

ing eyepiece is obtained from measurements of direct photographs taken at the Newtonian focus of the 100-inch telescope. If the nebula is at a considerable distance from the star, there is always the possibility that an undetermined amount of light may be lost by imperfect centering of the object on the slit of the spectrograph. This is true for the object observed in the cluster Ursa Major No. 2; the uncertainty in this respect is in this case greater than usual.

The nebular spectra shown in Plate III are direct enlargements of spectrograms made with the two-prism dispersion. The comparison

TABLE III

Type	Spectrum	Number
E0–7	G3.6	66
Sa	G3.4	30
Sb	G1.6	21
Sc	F8.8	19

spectra are of helium, the line farthest to the left being λ 3889, and to the right, λ 5016. The arrows, which point to the H and K lines, indicate the amount of the red-shift. The direct photographs (on the same scale and with approximately the same exposure-times) illustrate the decrease in size and brightness with increasing velocity or red-shift. NGC 4473 is in the Virgo Cluster; NGC 379 is a member of a group of nebulae in Pisces.

THE VELOCITY-DISTANCE RELATION FOR CLUSTERS

Figure 2 shows the velocity-distance relation for clusters[7] with the new data included. As faint members of the most distant clusters are below the limit of the 100-inch reflector, logarithms of the velocities have been plotted against the apparent photographic magnitude of the fifth brightest nebula in each cluster. Numbers in parentheses opposite the names of the clusters represent the number of nebulae observed in each cluster. Magnitudes in the Virgo Cluster were determined by Stebbins and Whitford with a photoelectric photometer; in the two clusters in Ursa Major, from extra-focal measures by

[7] The velocity-distance relation for isolated nebulae has been published by Hubble and Humason in *Mt. W. Comm.*, No. 116; *Proc. Nat. Acad.*, **20**, 264, 1934.

Baade; and in the remaining clusters, from extra-focal and schraffierkassette measures by Hubble. The agreement between different observers and the different methods employed indicates there is no serious systematic difference in the magnitude scale.

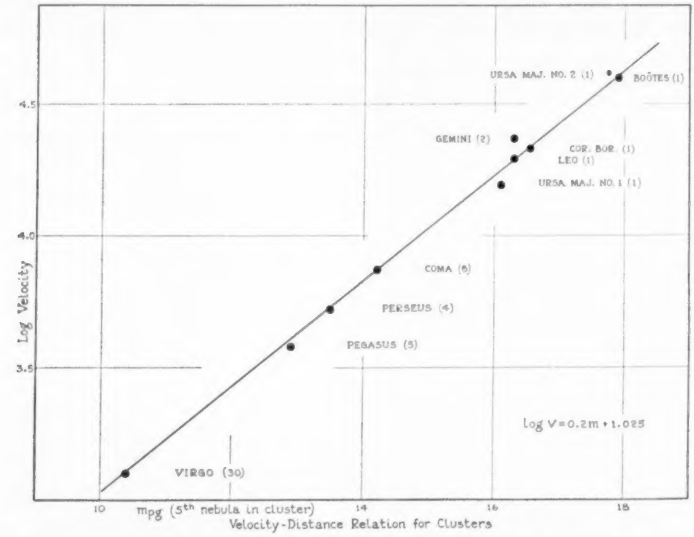


FIG. 2.—Figures in parentheses following the names of the clusters indicate the number of nebulae observed in each cluster.

As previously stated, the last well-determined velocity is that from the Boötes Cluster and indicates that the relationship is still sensibly linear to a distance of 70 million parsecs.

CARNEGIE INSTITUTION OF WASHINGTON
MOUNT WILSON OBSERVATORY
October 1935

THE MASS OF THE VIRGO CLUSTER*

SINCLAIR SMITH

ABSTRACT

The lists of radial velocities now include results for thirty-two members of the Virgo Cluster, thus giving for the first time sufficient data to determine some of the physical characteristics of a cluster of nebulae.

A comparison of the velocities of fainter members of the cluster with those of brighter members shows that the line-of-sight velocity of a nebula has no dependence on its magnitude; hence, equipartition apparently does not hold in the cluster. The distribution of the velocities in right ascension and declination shows that the cluster is not in rotation and that there is no central concentration of high velocities. This result is taken to mean that the cluster is neither condensing nor breaking up, but is a fairly stable assemblage, more or less held together by its gravitational field.

From the observed distribution function for radial velocity is derived the distribution function for space velocity. For an assumed distance of 2×10^6 parsecs this function leads to 2×10^{47} g or 10^{14} ⊙ as a value of the mass of the cluster. On the basis of 500 nebulae in the cluster, the mass per nebula is 2×10^{11} ⊙.

Although far larger than Hubble's value of 10^9 ⊙ for the mass of an average nebula, other evidence lends support to the high value obtained from the Virgo Cluster. It is possible that both figures are correct and that the difference represents a great mass of internebular material within the cluster.

Masses of extra-galactic nebulae have, in a few favorable cases, been derived from a study of relative line-of-sight velocities of different parts of the nebula. Similarly, we can turn to larger-scale phenomena and, by studying the relative line-of-sight velocities of different members of a cluster of nebulae, derive the mass of the cluster. In the first instance we determine the mass within the luminous boundaries of a nebula, while in the second we derive the total mass per nebula, including any internebular material within the cluster. Our present ideas concerning the transparency of space would suggest the presence of only a negligible amount of internebular material, and thus we should expect the two results to be more or less similar. Actually a discrepancy appears. In the following paragraphs we shall find that, applied to the Virgo Cluster, the second procedure gives a mass per nebula far larger than the mass of single nebulae as determined by Hubble.

Before considering the case of the Virgo Cluster, it is perhaps well to point out that both procedures suffer from the serious limitation

* *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington,* No. 532.

that we have no means of determining nebular accelerations. Our observations are restricted to the measurement of velocities, and hence in attempting to determine masses we must always assume that our structure is stable. As an example, suppose that a nebula consisting of a lenticular assembly of stars is set in uniform rotation. If it is observed within, say, 10^7 years, it will still appear to be in solid-body rotation. Actually, very different conditions will hold. Particles at a particular distance from the center will continue to describe circular orbits; particles within this distance and immediately outside will describe elliptical orbits; while the outermost particles will travel in either parabolic or hyperbolic orbits and will soon (say in 10^9 years) escape from the cluster. If the assembly is observed during the early part of its history, observations of rotation alone obviously will leave the mass completely indeterminate.

When we consider a cluster of nebulae, an additional uncertainty arises, namely, that the cluster may be only a statistical fluctuation in the space density of nebulae. In this case gravitational forces would play an insignificant part in the formation of the cluster, and the internal velocities would be unrelated to its mass. If, however, the cluster represents merely a statistical fluctuation, the cluster should appear to be either condensing or else evaporating into the surrounding space. In the detailed discussion of the Virgo Cluster which follows, it will be shown that, as far as can be determined, this particular cluster seems to be a stable assembly, held together by its own gravitational field. If this can be established, then the radial velocities now known should yield a satisfactory estimate for the mass of the cluster.

The material used consists of twenty-five radial velocities taken from Humason's lists and five by V. M. Slipher.[1] Nine of the fainter members were observed and measured by the writer, both to improve the sampling, since only bright members of the cluster had previously been observed, and to eliminate the possibility of a depend-

[1] *Mt. W. Contr.*, Nos. 426, 531; *Ap. J.*, **74**, 35, 1931; **83**, 10, 1936. Slipher's results are given by Strömberg in *Mt. W. Contr.*, No. 292; *Ap. J.*, **61**, 353, 1925. Humason's lists include twenty-seven velocities, but two are recent additions not used in the calculations reported here.

ence of the apparent radial velocity on magnitude.[2] The detailed discussion follows.

VELOCITY AS A FUNCTION OF CLUSTER CO-ORDINATES

The data were first examined for cluster rotation. Figure 1 shows the distribution of velocities across the cluster in right ascension and declination. It is evident that, if rotation is present, it must be very

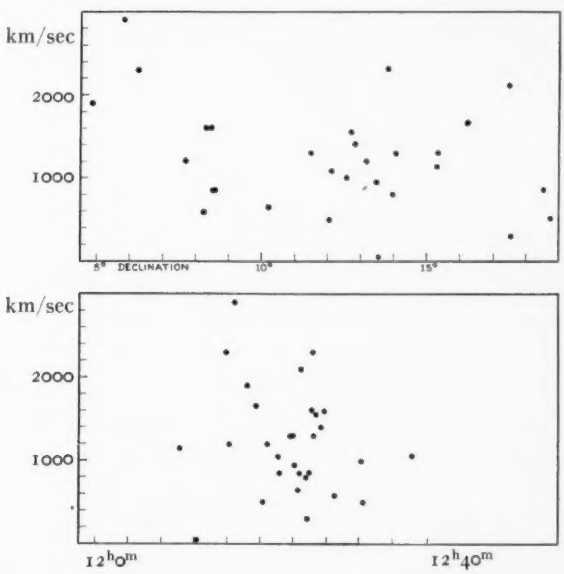


FIG. 1.—Distribution of velocities in declination and right ascension

small; and—more important for our purpose—there is no accumulation of high velocities toward the center of the cluster. A somewhat better demonstration of this point is furnished by Figure 2, in which the velocities are plotted against distance from the center of the cluster (center, $12^h\ 25^m$, $+12^\circ 30'$, 1930). Velocities of all values are more or less uniformly distributed over the range; and since this range covers the main portion of the cluster, we conclude that high line-of-sight velocities are just as likely to be found in one part of the

[2] No magnitude effect was found; and, as a corollary, there is no indication of equipartition of energy in the cluster. The mean line-of-sight velocity of the nine fainter nebulae is 1100 ± 200 km/sec, while that of the whole group is 1225 ± 330 km/sec.

cluster as another. If the cluster were either rapidly condensing or evaporating into space, a pronounced grouping of high velocities should occur near the center of the cluster. Since no such grouping

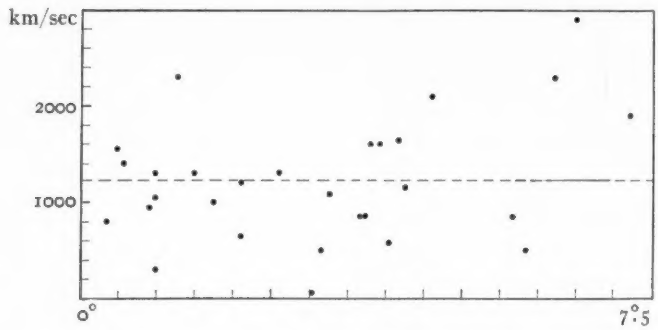


FIG. 2.—Distribution of velocities as a function of distance from center of the cluster.

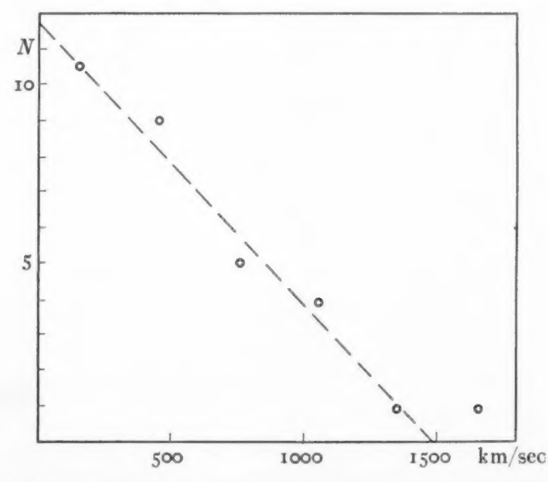


FIG. 3.—Distribution function of cluster velocities. Ordinates are numbers of nebulae for a velocity interval of 300 km/sec.

appears, it seems reasonable to assume that we are dealing with a more or less stable structure. If the orbits of the individual nebulae were largely circular, and random distribution of the orbital planes is assumed, we should expect the line-of-sight velocities to decrease systematically as we approach the center of the cluster. On the other hand, if the orbits were in the main highly elliptical, we should ex-

pect just the reverse radial distribution. The observed distribution agrees well enough with the hypothesis that we are dealing with a mixture of both types of orbits, and that orbits of all eccentricities are to be found in the cluster.

THE LINE-OF-SIGHT VELOCITY DISTRIBUTION FUNCTION

The line-of-sight velocity distribution for the whole cluster is plotted in Figure 3. In this case the mean velocity (1225 km/sec) has been subtracted from each observed value, and the difference (without regard to sign) has been taken to be the peculiar velocity of the individual. These velocities are grouped in intervals of 300 km/sec. The dotted line is drawn in as a possible representation of the distribution function.

If we assume random distribution for the orbits, Figure 3 tells us that the actual space velocities in the cluster probably cover the entire range from zero to a maximum of about 1500 km/sec, for, if the space velocities were more or less the same for all nebulae, we should expect a very different distribution function. For example, in the case of space velocities having the same magnitude but random distribution, the number dN_v showing line-of-sight velocities between v and $v+dv$ (considering only a hemisphere), would be

$$dN_v = N_u \frac{2\pi U dv}{2\pi U^2} = \frac{N_u}{U} dv \,.$$

Hence, all line-of-sight velocities between 0 and V would have the same frequency. In this case the velocity distribution-curve would be a horizontal straight line between zero and the maximum velocity, 1500 km/sec.

If we assume that the number of observed line-of-sight velocities varies linearly with the observed velocity as shown in Figure 3, i.e.,

$$dN \approx c\left(1 - \frac{v}{U}\right) dv \,, \tag{1}$$

the actual space velocity distribution function can be derived as follows:

Let $\phi\ (u) du$ be the number of nebulae having a velocity between u and $u+du$ and randomly distributed in direction over the hemi-

sphere $\theta=0$ to $\pi/2$, $\phi=0$ to 2π. For the number of nebulae with velocities between u and $u+du$ which lie in the range v to $v+dv$, we can write

$$dN=\frac{\phi(u)}{u}\,dudv\ .$$

For the total number in range dv we then get

$$dN=dv\int_v^U \frac{\phi(u)}{u}\,du\ .$$

But observationally we have equation (1) as an expression for dN, and by comparison obtain

$$\int_v^U \frac{\phi(u)}{u}\,du=c\left(1-\frac{v}{U}\right)\ .$$

By differentiating with respect to the lower limit,

$$-\frac{\phi(u)}{u}=-\frac{c}{U}\ ,$$

or

$$\phi(u)=\frac{c}{U}\,u\ , \tag{2}$$

between $u=0$ and $u=U$.

Since $\phi\ (u)$ increases with u to a sharp cut-off at $u=U$, it is reasonable to regard U as the velocity of escape from the cluster.

If we had assumed a Gaussian law for the line-of-sight distribution function, the transformation to space velocities would have had the usual form; but in this case both distribution functions would extend to infinity, and hence would not be suitable for our purpose. Probably the actual space velocity distribution function in the cluster lies between equation (2) and a Gaussian distribution-curve.

In principle, a further extension of the analysis is possible, since the velocity distribution function for any small region in the cluster will depend both on the distance of the region from the center and on the density distribution in the cluster. Practically, such an exten-

sion is not justified at present, since the observations are not now sufficient to establish the form of the distribution function even for the cluster as a whole. One can only say that the observations are compatible with a linear distribution function for radial velocities which, by equation (2), makes the space velocity distribution function more or less linear.

THE MASS OF THE CLUSTER

In spite of the fact that the foregoing linear distribution function can be regarded as only a first approximation, it does show with considerable certainty that cluster velocities up to 1500 km/sec must be fairly common. In accordance with the preceding discussion, we should adopt this value as the velocity of escape; but since Figure 2 shows some grouping of high velocities at large distances from the center, it is also reasonable to assume that the outermost particles move in circular orbits with a speed of 1500 km/sec. Hence we can write either

$$m=\frac{v^2r}{2G} \quad \text{or} \quad \frac{v^2r}{G} ,$$

the difference being of small importance.

Taking the circular orbit form, and assuming for radius of the cluster 2×10^5 parsecs (i.e., 0.1 times its distance), we find for the mass

$$2\times 10^{47} \text{ grams} \quad \text{or} \quad 10^{14} \odot .$$

Assuming 500 nebulae in the cluster and no internebular material, we find for the mean mass of a single nebula

$$4\times 10^{44} \text{ grams} \quad \text{or} \quad 2\times 10^{11} \odot .$$

This value is some two hundred times Hubble's[3] estimate of $10^9 \odot$ for the mass of an average nebula. The cause of the discrepancy is not clear. In the determination of the mass of the cluster, the only source of a large error in the result is a possibility already mentioned, namely, that the cluster is simply a statistical fluctuation in space density. The extremely small probability of such an occurrence, to-

[3] *Mt. W. Contr.*, No. 485; *Ap. J.*, **79**, 8, 1934.

gether with the evidence already presented, seems to rule out this possibility.

On the other hand, the view that the cluster possesses a powerful gravitational field is strongly supported by the fact that the mean peculiar velocity of cluster nebulae is about four times the 150 km/sec found by Hubble[4] for isolated nebulae.[5] We can hardly interpret this fact in any other way than that nebulae leaving a cluster lose energy and nebulae captured by a cluster gain energy.

A consideration of other groups offers additional support. The Coma Cluster, which resembles the Virgo Cluster, shows a similar range in velocity,[6] while the small group to which our galaxy belongs shows a very small range. We therefore conclude from the available evidence that our procedure is justified and that the mass derived for the Virgo Cluster should be approximately correct.

The difference between this result and Hubble's value for the average mass of a nebula apparently must remain unexplained until further information becomes available. A statistical study of the relative velocities of close pairs of nebulae may possibly furnish the required data. It is also possible that both values are essentially correct, the difference representing internebular material, either uniformly distributed or in the form of great clouds of low luminosity surrounding the nebulae, as suggested by the recent great extension of the boundary of M 31.[7] Whatever the correct answer, it cannot be given with certainty at this time.

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September 1935

[4] *Mt. W. Comm.*, No. 105; *Proc. Nat. Acad.*, **15**, 168, 1929.

[5] I am indebted to Dr. Hubble for suggesting this point.

[6] F. Zwicky has pointed out (*Helv. Phys. Acta*, **6**, No. 2, p. 110, 1933) that the velocity range in the Coma Cluster indicates non-luminous matter which is some four hundred times the amount of the observed luminous material.

[7] Stebbins, *Mt. W. Comm.*, No. 113; *Nat. Acad. Proc.*, **20**, 93, 1934.

THE SPECTRA OF EARLY-TYPE STARS IN THE NEAR ULTRA-VIOLET REGION

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ABSTRACT

The spectra of three B stars and of eleven A stars have been measured for wavelengths in the region 3570–3930 A. Identifications are made for most of the lines, and a discussion of the behavior of the elements is given.

The spectrograms used in this investigation were obtained with the one-prism Yerkes spectrograph, which was attached to the 69-inch reflector of the Perkins Observatory. Most of the plates could be measured as far into the violet as λ 3680; one plate of α Cygni was measured as far as 3570 A. The scale of the spectrograms is 15.4 A per millimeter at λ 3780. For the reduction of the measures quadratic formulae were used.

The stars investigated, together with their Harvard spectral types, are given in Table I.

TABLE I

γ Pegasi B2	θ Aurigae Aop
γ Orionis B2	α Canum Venaticorum Aop
β Orionis B8p	ϵ Ursae Majoris Aop
α Lyrae Ao	θ Leonis Ao
γ Geminorum Ao	α Geminorum Ao
α Cygni A2p	α Canis Majoris Ao
α Andromedae Aop	γ Lyrae Aop

For the identifications Miss Moore's table[2] was found to be invaluable; Kaiser's tables and the *Revised Rowland* were also used.[3]

In the case of the B stars, a number of lines are added to Marshall's[4] and to Struve's[5] lists and the evidence for the identification of P^{++} and Al^{+} is strengthened. About 60 per cent of the lines may be considered as fairly well identified.

The only extended investigation of stellar spectra of type A in the

[1] Advanced fellow of the C.R.B. Educational Foundation.

[2] *A Multiplet Table of Astrophysical Interest* (Princeton, 1933).

[3] For Fe^{+} the new measures by Dingle have been used (*M.N.*, **95**, 704, 1935).

[4] R. K. Marshall, *Pub. Obs. U. of Michigan*, **5**, No. 12, 1934.

[5] O. Struve, *Ap. J.*, **74**, 225, 1931.

near ultra-violet region is that of α Cygni by Wright.[6] Table III includes in the common wave-length region many lines additional to those measured by him.

The results are collected in Tables II and III, which deal, respectively, with the B and A stars. In the last column of Tables II and III the figures in parentheses are the Rowland intensities in the solar spectrum; the figures in brackets are laboratory intensities. The identifications in braces are doubtful.

In Table III the second column contains the mean wave-lengths and intensities for the following stars: α Lyr, γ Gem, θ Leo, α Gem, α CMa, and γ Lyr. These six spectra are very similar and have not been listed separately. The identifications listed in the last column of Table III cannot be considered as complete for every case; many of them are satisfactory, but for a number it is certain that other contributors will have to be considered. The identifications are probably about as complete as they can be made at this time.

The behavior of the elements in the different spectra agrees quite well, for the normal stars, with Morgan's description.[7]

EXAMINATION OF THE INDIVIDUAL ELEMENTS[8]

H. The Balmer lines are usually visible as far as $H\omega(=H_{26}=3667.87$ A).

He. The strongest line in this region is 3819.61 [4], it is either missing in A spectra or blended with the red part of 3820.61. The fainter lines 3926.53 [1], 3867.54 [2+1], and 3838.09 [1], which coincide with observed absorption lines, are presumably chance coincidences.

C^+. The strongest lines in this region are 3918.98 [6] and 3920.68 [8]; they probably contribute very little to any A-type spectrum.

O^+. The two lines 3919.28 [6] and 3911.95 [10], which are fairly strong in the B stars, are merely chance coincidences in the A stars; the other four lines, 3875.82 [4], 3864.45 [5], 3830.45 [4], and 3727.33 [8], are also chance coincidences, as they do not appear even in β Orionis (B8).

6 W. H. Wright, *Lick Obs. Bull.*, No. 332, 1921.

7 W. W. Morgan, *Pub. Yerkes Obs.*, **7**, Part 3, 1935.

8 The elements indicated here are only those for which the spectral region is favorable.

TABLE II

Approximate Wave-Length	γ Peg B2	γ Ori B2	β Ori B8p	Identifications
3704	4.97(1)			*He* 5.00[3]; *He* 5.14[1]; $H\xi$ 386; O^{++} 4.73[3]
3727	7.28(1)			O^{+} 7.33[8]; Ne^{+} 7.08[9]
3728	8.67(?1)			O^{++} 8.70[1]
3734	4.30(H)		4.25(H)	$H\lambda$ 4.37
3737			7.77(1)	Al^{+} 8.00[3]
3750	0.21(H)		0.12(H)	$H\kappa$ 0.15
3758			8.98(4)	Ti^{+} 9.29[200]
3761			1.31(3)	Ti^{+} 1.32[200]
3770	0.60(H)		0.62(H)	$H\iota$ 0.63
3775	5.90(2)			
3777	7.39(1)			O^{+} 7.60[4r]; Ne^{+} 7.16[8]
3780	0.14(1)			
3781	1.81(1)			
3787	7.69(4)			*He* 7.49[1]
3791	1.38(1)			Si^{++} 1.41[3]; O^{++} 1.26[6]
3794	4.43(1)			O^{+} 4.48[32]; S^{++} 4.65[1]
3796	6.24(1)	6.40(1)		Si^{++} 6.11[4]
3797	7.95(H)	7.97(H)	7.85(H)	$H\theta$ 7.90; Ne^{+} 0.02[5]
3802	2.59(2)			
3802	2.94(3)			O^{+} 3.14[6r]
3806	6.62(7)	6.58(10)	6.17(4)	Si^{++} 6.56[5]; *He* 5.75[1]
3807	7.62(1)			
3809	9.46(1)	9.02(1)		
3810	0.97(1)	0.59(1)		
3812	2.23(1)	1.89(1)		
3813	3.20(1)			
3813	3.84(1)			
3814	4.95(0?)			
3815	5.72(1)			
3817	7.06(15)			
3819	9.89(9)	9.53(9)	9.87(10)	*He* 9.61[4]; *He* 9.75[3]
3826		6.36(1)		
3833	3.21(1)			*He* 3.56[1]; O^{+} 3.10[3]
3835	5.43(H)	5.62(H)	5.59(H)	$H\eta$ 5.39
3840	0.36(1)			
3846	6.95(0?)			?O^{+} 7.89[3]; N^{+} 7.38[3]?
3850	0.07(1)	9.98(1)		
3853	3.22(1)	3.37(1)	3.68(4)	Si^{+} 3.66[3]
3855	5.88(3)	5.64(2)	6.15(7)	Si^{+} 6.02[8]; O^{+} 6.16[5]; N^{+} 6.07[3]
3857	7.27(1)			O^{+} 7.18[4]
3858	8.38(0)			
3858	8.83(1)	9.69(?2)		
3862	2.79(2)	2.83(1)	2.68(5)	Si^{+} 2.59[6]
3864	4.80(1)	4.80(1)		O^{+} 4.45[5]; O^{+} 4.68[1]
3867	7.51(6)	7.76(4)	7.20(3)	*He* 7.46[2]; *He* 7.62[1]
3870	0.11(0)			
3872	2.23(3)	1.99(3)	2.00(2)	*He* 1.80[1]
3876	6.49(4)	6.71(2)		C^{+} 6.19[4]; C^{+} 6.41[2]; O^{+} 5.82[4]; O^{+} 6.05[1]; C^{+} 6.67[1]
3878	8.39(1)	8.37(1)		*He* 8.18[1]
3880		0.13(1)		C^{+} 0.59[1]; C^{+} 9.60[1]

TABLE II—*Continued*

Approximate Wave-Length	γ Peg B2	γ Ori B2	β Ori B8p	Identifications
3882	2.09(4)	2.38(3)	...	O^+ 2.20[7]; O^+ 2.45[1]
3884	...	4.07(0?)	...	...
3888	...	8.58(?)	...	...
3888	8.94(H)	8.98(H)	8.80(H)	*Hξ* 9.05; *He* 8.65[10]
3889	...	...	9.97(1)	...
3891	...	1.58(1)	1.21(1)	...
3892	2.44(0)	...	...	S^+ 2.32[5]
3893	...	3.73(1)	3.70(1)	O^+ 3.52[2]
3895	5.29(1)	...	5.50(1)	P^{++} 5.03[6]
3896	...	6.31(?)	...	?O^+ 6.30[1]?
3897	...	7.85(?)	...	...
3898	...	...	8.71(1)	...
3899	9.43(1)	9.79(1)	...	...
3900	...	...	0.81(1)	Ti^+ 0.55[50]; Al^+ 0.68[10]
3903	3.46(1)	3.66(0)	3.50(0)	...
3904	4.81(1)	...	...	P^{++} 4.79[6]
3905	...	...	5.93(?)	...
3907	7.74(1)	7.80(2)	...	O^+ 7.45[4]
3908	...	...	8.69(1)	...
3910	9.85(?)	...	0.63(?)	...
3912	2.18(4)	2.47(3)	2.19(1)	O^+ 1.96[10]; O^+ 2.09[2]
3913	...	...	3.56[0]	...
3914	4.41(0)	...	...	...
3914	4.87(2)	4.63(2)	...	...
3915	...	...	5.70(0)	...
3917	7.30(2)	6.79(1)	...	...
3918	8.88(5)	9.30(4)	8.79(4)	C^+ 8.98[6]; O^+ 9.29[6]; N^+ 9.00[6]
3920	0.49(5)	0.75(4)	0.40(4)	C^+ 0.68[8]
3921	1.74(0)	...	2.48(1)	...
3923	3.14(1)	...	...	S^+ 3.47[4]; P^{++} 2.72[4]
3924	4.30(3)	4.54(3)	4.06(0)	Si^{++} 4.44[4]
3926	6.66(4)	7.07(4)	6.42(4)	*He* 6.53[1]
3927	...	...	7.93(1)	...
3928	8.78(3)	8.57(2)	...	S^{++} 8.59[5]
3929	...	...	9.64(1)	...
3931	...	1.78(1)	...	S^+ 1.88[2]

Ne^+. The absence of 3777.16 [8], which would be in a favorable region, seems to be strong evidence against the identification of Ne^+; this agrees with the expected maximum intensity in the B stars. All the other coincidences are probably due to chance.

Mg. The triplet 3838.30 [100r], 3832.31 [80r], 3829.37 [40] is very strong in all the A stars except in α And and θ Aur, where the three lines are missing. The triplet is especially strong in α Lyr, α Gem, ε UMa, γ Gem, and α CMa. Usually the observed estimated

TABLE III

Approx. Wave-Length	Ordinary Ao	α Cyg	α And	θ Aur	α^2 CVn	ε UMa	Identifications
3572		2.66(2)					*Fe* 2.02(5); Sc^+ 2.52(4+6)
3576		6.66(2)					*Fe* 5.98(4); Sc^+ 6.36(5); *Ni* 6.66[3]
3581		1.24(2)					*Fe* 1.21(30); Sc^+ 0.93(5)
3583		3.67(1)					*Fe* 3.34(5)
3585		5.56(4)					*Fe* 5.71(6); *Fe* 5.34(7); Cr^+ 5.52(2)
3587		7.15(2)					*Fe* 7.23(7); *Fe* 6.99(8); Al^+ 7.06[8]; Al^+ 6.55[10]; {Ti^+7.15(2)}
3603		3.69(2)					*Fe* 3.83(4); Cr^+ 3.78(3); Cr^+ 3.62(2); *Fe* 3.21(5)
3609		9.00(1)					*Fe* 8.87(20)
3619		9.08(1)					*Fe* 8.78(20); *Ni* 9.40(8)
3621		1.08(3)					*Fe* 1.47(6); Fe^+ 1.27(0); {*V* 0.97(−1)}
3623		3.41(1)					*Fe* 3.19(5); *Fe* 3.79(4)
3624		4.63(3)					*Fe* 5.15(5); *Fe*–Ti^+ 4.84(5); *Fe* 4.30(3); Fe^+4.89[3]
3630		0.42(1)					*Fe* 0.36(4); ⊙ 0.74(4); Sc^+ 0.74(3); {Zr^+0.03(1)}
3632		2.38(4)					*Fe* 2.05(3); *Fe* 2.56(3); *Fe* 1.48(15); Cr^+ 1.71(1); Fe^+ 2.29(−1)
3635		5.85(1)					*Fe* 6.17(3); *Fe* 6.24(2); *Fe* 5.47(4)
3639		9.79(1)					*Fe* 0.39(6)
3641		1.12(1)					Ti^+ 1.34(4)
3642		2.83(1)					Sc^+ 2.83(3); {*Ti* 2.68(7)}
3643		3.25(1)					*Fe* 3.63(4)
3667		7.68(H)					$H\omega$ 7.68
3669		9.47(H)					$H\psi$ 9.47
3671		1.48(H)					$H\chi$ 1.48
3677		7.10(H)				7.13(H)	$H\upsilon$ 6.36; *Fe* 6.32(6); *Fe* 6.88(3); *Fe* 7.32(4)
3678	8.05(12)	7.85(3)			8.12(10)		Cr^+ 7.86(3); Cr^+ 7.70(3); *Fe* 7.63(5)
3679	9.60(H)	9.70(H)			9.70(H)	9.66(H)	$H\tau$ 9.36; *Fe* 9.92(9)
3682	3.00(H)	3.07(H)			2.85(H)	2.80(H)	$H\sigma$ 2.81
3684					4.16(4)		*Fe* 4.12(7)
3685	5.52(5)	5.30(4)			5.66(7)	5.24(7)	Ti^+ 5.20(10)
3686	6.81(H)	6.96(H)			6.63(H)	6.71(H)	$H\rho$ 6.83

TABLE III—*Continued*

Approx. Wave-Length	Ordinary Ao	α Cyg	α And	θ Aur	a^2 CVn	ε UMa	Identifications
3691	1.60(H)	1.58(H)			1.51(H)	1.46(H)	*Hπ* 1.56
3697	7.21(H)	7.11(H)			7.16(H)	7.27(H)	*Ho* 7.15
3703	3.89(H)	3.91(H)			3.77(H)	3.67(H)	*Hξ* 3.85
3706	6.31(6)	6.45(3)				5.90(—)	Ti^+ 6.22(3); *Fe* 5.58(9); Ca^+ 6.04(6)
3708	8.05(—)				8.41(4)	8.12(—)	*Fe* 7.83(5); *Fe* 7.93(5)
3709	9.98(4)				9.62(5)	9.70(—)	*Fe* 9.26(8); Y^+ 10.29[500]
3710	11.06(—)				10.65(4)		*Fe* 11.23(4); *Fe* 11.41(3)
3712	2.05(H)	2.04(H)			2.29(H)	2.18(H)	*Hν* 1.97
3712		2.99(2)			3.41(—)		Cr^+ 2.94(3)
3715	5.77(6)	4.94(4)			5.55(8)	5.59(6)	Cr^+ 5.18(2); *Fe* 5.92(3); {V^+ 5.48(4)}
3717	6.93(3)				7.50(1)		
3718	8.24(3)				8.63(2)		*Fe* 8.41(4)
3719	0.16(1)	9.77(2)			9.82(—)	9.90(—)	*Fe* 9.95(40)
3721	2.06(H)	2.09(H)			1.79(H)	1.88(H)	*Hμ* 1.94
3723					3.14(—)		*Fe* 2.59(6)
3725					5.98(?)		*Fe* 5.50(3)
3727	7.45(5)	7.34(3)			7.18(6)	7.31(4)	*Fe* 7.10(3); V^+ 7.35(1); {Zr^+ 7.69(1)}
3729	9.92(3)						*Fe* 0.39(3)
3731	1.31(4)						*Fe* 1.38(3); *Fe* 0.95(3)
3732	2.82(0)	2.92(1)				2.35(?)	*Fe* 2.41(6); V^+ 2.75(2)
3734	4.67(H)	4.54(H)			4.93(H)	4.73(H)	*Hλ* 4.37; *Fe* 4.88(40)
3737	7.34(8)	7.13(3)			7.55(6)	6.97(4)	*Fe* 7.14(30); Ca^+ 6.92(5)
3738	8.60(3)	8.25(1)		8.39(—)	8.84(3)	8.78(4)	*Fe* 8.31(3); Cr^+ 8.36(2)
3740	0.17(2)	0.40(0)			0.36(4)	0.28(?)	*Fe* 0.25(3)
3741	1.72(5)	1.77(8)		1.60(5)	1.87(4)	1.67(4)	Ti^+ 1.65(4)
3743	3.47(4)		3.25(—)	3.54(5)	3.54(5)	3.55(5)	*Fe* 3.37(6)
3744	4.63(2)						
3745	5.69(6)	5.67(2)		5.59(—)	5.59(5)	5.57(5)	*Fe* 5.58(8); *Fe* 5.91(6); Zr^+ 5.97(6?)
3746						6.84(—)	*Fe* 6.92(3)
3748	8.45(4)	8.46(6)	8.82(—)		8.01(6)	8.34(—)	*Fe* 8.27(10); Cr^+ 8.68(1)
3749	9.60(1)						*Fe* 9.50(20)

TABLE III—*Continued*

Approx. Wave-Length	Ordinary Ao	α Cyg	α And	θ Aur	α^2 CVn	ε UMa	Identifications
3750	0.11(H)	0.18(H)	0.17(H)	0.09(H)	0.01(H)	0.06(H)	*Hκ* 0.15
3752	2.94(—)					2.97(—)	
3753	3.63(—)				3.82(—)		*Fe* 3.62(6?)
3754	4.80(—)	4.48(2)			4.78(5)	4.88(5)	Cr^+ 4.58(1); *Fe* 4.51(3); Cr^+ 5.14(0)
3755		5.15(1)					Cr^+ 5.14(0)
3755			5.89(4)	6.27(?)	5.82(—)		*Fe* 6.07(3)
3756					6.74(4)		*Fe* 6.94(4)
3757	7.80(4)		7.69(3)	7.88(—)		8.09(3)	Ti^+ 7.69(4)
3758	8.47(4)	8.51(3)			8.49(5)		*Fe* 8.25(15)
3759	9.59(5)	9.20(10)	9.86(4)	9.66(4)	9.67(6)	9.49(4)	Ti^+ 9.30(12); Fe^+ 9.46(0)
3761	1.60(6)	1.19(10)	1.92(0)	1.78(0)	2.12(6)	1.77(6)	Ti^+ 1.32(7); Ti^+ 1.88(3)
3762	2.22(5)						Ti^+ 1.88(3); {*Fe* 2.21(2)}
3763	3.15(1)	3.09(1)					Fe^+ 2.89(−1)
3763				3.75(5)	3.48(4)		*Fe* 3.80(10)
3764	4.44(5)	3.98(2)	4.41(5)		4.59(5)	4.20(4)	
3765	5.81(4)	5.80(1)		5.47(0)	6.02(4)	5.89(4)	*Fe* 5.55(6)
3767	7.35(4)	7.12(1)	7.66(−)		7.17(—)	7.45(—)	*Fe* 7.21(8); *Fe* 6.67(3)
3767					7.74(—)		
3768	8.44(1)						*Fe* 8.04(3)
3769	9.66(—)	9.64(8)			9.19(—)		Ni^+ 9.46(3)
3770	0.68(H)	0.66(H)	0.89(H)	0.57(H)	0.73(H)	0.63(H)	*Hι* 0.63
3773	3.45(—)		3.46(—)		2.97(—)	3.15(6)	*Fe* 3.70(3)
3774	4.42(—)				4.37(7)	4.43(6)	Y^+ 4.34(3); *Fe* 4.83(4)
3776	6.08(—)	6.25(1)	5.46(—)	5.74(0)	6.08(6)	6.21(5)	Ti^+ 6.06(2); *Fe* 6.46(3)
3778	8.32(?)		8.00(—)		8.02(5)	7.93(6)	*Fe* 8.33(3)
3778					8.81(5)		*Fe* 8.71(3)
3779	9.86(?)	9.43(1)			9.61(4)		*Fe* 9.43(4)
3780			0.64(0)				⊙ 0.71(3)
3781	1.47(?)	1.37(1)			1.85(4)	1.26(3)	*Fe* 1.19(3); Fe^+ 1.51(−1)
3782					2.87(—)		
3783	3.39(4)	3.27(6)	3.79(0)		3.60(4)	3.53(4)	Fe^+ 3.35(2); *Ni* 3.54(6)

TABLE III—*Continued*

Approx. Wave-Length	Ordinary Ao	α Cyg	α And	θ Aur	α^2 CVn	ϵ UMa	Identifications
3784	4.40(0)					4.57(?)	
3786	6.12(3)				6.32(—)	6.04(5)	*Fe* 6.18(4); *Fe* 6.68(5); *Fe* 5.95(3)
3787	7.97(3)		7.87(0)		8.03(—)	7.74(4)	*Fe* 7.89(9)
3788	9.26(1)				8.80(5)		Y^+ 8.70(2); *Fe* 9.19(3)
3790					0.38(—)	0.19(3)	*Fe* 0.10(5)
3793					3.45(—)		
3795	5.20(—)				4.96(4)	5.33(4)	*Fe* 5.01(8); {La^+ 4.77(1)}
3796	6.03(?)						
3797	7.96(H)	8.08(H)	8.04(H)	8.01(H)	7.83(H)	8.03(H)	*Hθ* 7.90
3799	9.67(2)						*Fe* 9.56(7)
3801					1.53(—)		*Fe* 1.68(3); {Ce^+ 1.53[10]}
3805	5.39(—)				5.65(—)	5.14(5)	*Fe* 5.35(6)
3806	6.90(—)				6.94(3)	6.83(5)	*Fe* 6.72(8)
3809					9.49(3)		
3811					1.22(3)		{*Fe* 0.76(3)}
3813	3.07(3)		2.87(4)		2.50(2)	3.05(4)	*Fe* 2.96(5); Ti^+ 3.40(2)
3814	4.67(4)	4.28(3)	4.54(0)	4.36(5)	4.39(5)	4.50(4)	Ti^+ 4.60(3); *Fe* 4.52(4); Fe^+ 4.12(0)
3815					5.03(3)		
3816	6.03(4)	5.83(2)		5.96(—)	6.01(5)	6.09(4)	*Fe* 5.85(15)
3817	7.70(1)		7.48(0)		7.80(4)	7.54(3)	*Fe Ti* 7.65(3)
3819	9.18(1)				9.23(3)	9.03(0)	{Eu^+ 9.66[50]}
3820	0.61(6)	0.43(2)	0.77(0)	0.33(—)	0.58(5)	0.40(4)	*Fe* 0.44(4)
3821	1.65(2)				2.31(4)	2.25(3)	*Fe* 1.84(4)
3822		2.96(1)					
3823	3.63(—)						
3824	4.64(4)	5.01(2)		4.81(—)	4.84(5)	4.67(4)	*Fe* 4.45(6)
3825	5.89(5)		5.69(0)		5.43(5)		*Fe* 5.89(20)
3826	5.89(5)	6.03(1)			6.35(5)	6.14(4)	*Fe* 5.89(20)
3827	7.70(4)	7.21(1)	7.20(—)		7.50(4)		Fe^+ 7.08(0); *Fe* 7.83(8)
3828					8.09(4)	7.86(4)	*Fe* 7.83(8)
3829	9.36(4)	9.35(2)				9.58(4)	*Mg* 9.37(10)

TABLE III—*Continued*

Approx. Wave-Length	Ordinary A0	α Cyg	α And	θ Aur	α^2 CVn	ε UMa	Identifications
3830					0.35(—)		
3830	0.82(—)					1.10(3)	*Fe* 0.87(2); *Fe* 0.77(2)
3832	2.36(6)	2.44(4)				2.36(5)	*Mg* 2.31(15); Y^+ 2.89(3)
3833	4.05(—)					3.99(—)	*Fe* 4.23(10); *Fe* 3.87(3)
3835	5.43(H)	5.63(H)	5.57(H)	5.33(H)	5.20(H)	5.40(H)	$H\eta$ 5.39
3836		6.37(—)					Ti^+ 6.09(2); *Fe* 6.39(3)
3837			7.14(?)			7.12(?)	{*Fe* 7.14(2)}
3838	8.28(8)	8.42(3)			8.10(—)	8.22(7)	*Mg* 8.30(25)
3840	0.83(5)	0.96(2)	0.76(5)	1.10(?)	0.56(—)	0.76(5)	*Fe* 1.06(10); *Fe* 0.46(8)
3843	3.26(2)					3.31(3)	*Fe* 3.27(4); *Fe* 2.99(3); Sc^+ Zr^+ 3.06(2)
3844			4.46(5)	4.49(—)			
3845	5.32(3)	5.42(3)		5.58(4)		5.22(3)	*Fe* 5.18(3)
3846	6.72(2)				7.01(—)	7.11(3)	*Fe* 6.81(5)
3849		9.29(1)			8.92(—)		
3850	0.14(4)	0.56(4)		9.98(—)	0.36(—)	9.95(4)	*Fe* 9.98(10)
3851	1.04(1)		1.36(2)		1.39(—)		*Fe* 0.83(4)
3851		1.80(1)					
3852	2.70(2)						*Fe* 2.58(4)
3853	3.79(3)	4.01(4)	4.29(4)	3.96(6)	3.99(6)	3.74(2)	Si^+ 3.67(—1)
3856	6.37(7)	6.36(6)	6.55(5)	6.35(10)	6.52(8)	6.46(4)	Si^+ 6.03(1); *Fe* 6.38(8)
3857	7.86(2)			7.92(2)			
3858	8.85(2)						*Fe* 9.22(3); *Ni* 8.30(7)
3860	0.15(4)	0.12(2)	9.81(3)	0.21(4)	0.37(4)		*Fe* 9.92(20)
3861	1.47(1)				1.46(1)	1.44(4)	*Fe* 1.35(3)
3862	2.92(4)	2.96(5)	3.05(4)	2.89(10)	2.85(7)	2.65(2)	Si^+ 2.59(1)
3864	4.33(2)			4.47(?)	4.14(3)	3.97(2)	
3866	5.79(4)	5.87(2)			5.62(6)	5.89(4)	*Fe* 5.53(7)
3867	7.18(1)			6.89(10)		7.27(3)	*Fe* 7.23(3)
3868				8.32(2)			
3869				0.10(3)		9.60(?)	*Fe* 9.56(3)
3871					1.09(5)		

TABLE III—*Continued*

Approx. Wave-Length	Ordinary A0	α Cyg	α And	θ Aur	a^2 CVn	ε UMa	Identifications
3872	2.62(4)	2.58(2)	2.54(3)	2.64(4)			*Fe* 2.51(6)
3873	3.82(2)						*Fe* 3.77(4)
3874	5.06(1)			4.64(—)	4.41(2)		
3878	8.09(3)						*Fe* 8.03(8)
3878	8.61(4)	8.65(2)	8.37(4)	8.99(—)	8.50(4)	8.42(6)	*Fe* 8.58(7); *Fe* 8.68(7); V^+ 8.75[35]
3882	2.25(2)				2.37(—)		
3882	2.76(?)			3.19(—)			
3883	3.93(—)						
3884					4.71(—)		
3885	5.75(?)						*Fe* 5.52(4)
3886	6.55(3)					6.23	*Fe* 6.30(15); *Fe* 7.06(7)
3889	9.02(H)	9.22(H)	8.85(H)	9.13(H)	8.95(H)	9.02(H)	*Hξ* 9.05
3891	1.31(—)			1.66(7)	2.09(—)		*Fe* 1.94(4); {Ba^+ 1.78(0)}
3893						3.20(—)	*Fe* 3.40(4)
3895	5.68(3)			5.43(—)	5.50(1)	5.99(—)	*Fe* 5.67(7)
3896				6.54(—)			
3897	7.75(2)		7.62(4)	8.33(—)		7.93(4)	*Fe* 8.02(5); *Fe* 7.90(3)
3898					8.84(3)		*Fe* 9.04(3)
3899	9.78(2)						*Fe* 9.72(8)
3900	0.71(4)	0.81(4)	0.61(4)	0.48(4)	0.79(4)	0.50(4)	Ti^+ 0.54(5); Al^+ 0.66[10]
3902	2.53(4)		2.61(4)		2.78(2)		*Fe* 2.96(10)
3903	3.06(4)			3.05(5)		3.27(4)	*Fe* 2.96(10)
3903	3.68(4)	3.62(1)			4.11(2)		*Fe* 3.92(5); V^+ 3.26[25]
3905	5.93(4)	5.82(4)	5.94(4)	5.97(7)	5.85(6)	6.44(4)	*Fe* 6.49(10); *Si* 5.53(8); Fe^+ 6.04(−1)
3907				7.51(4)	7.53(4)		Eu^+ 7.11[300R]; *Fe* 7.94(5); *Sc* 7.48(3)
3908	8.48(0)				8.61(3)		
3909	9.36(2)						*Fe* 9.67(4)
3910			0.69(?)			0.05(3)	{*Fe* 0.85(4)}; *Fe* 9.67(4)
3911				2.02(4)	1.63(5)	1.58(3)	*Sc* 1.83(2)
3913	3.66(4)	3.43(4)	4.05(2)		3.85(4)	3.64(4)	Ti^+ 3.47(5); *Fe* 3.64(4)
3914	4.74(4)	4.57(2)		4.69(4)	4.84(4)	4.84(4)	*Fe* 4.29(3); V^+ 4.57[20]

TABLE III—*Continued*

Approx. Wave-Length	Ordinary A0	α Cyg	α And	θ Aur	α^2 CVn	ε UMa	Identifications
3916	6.58(3)	.....	.....	.....	.....	6.61(2)	*Fe* 6.74(5); V^+ 6.41[20]
3917	7.27(3)	.....	7.24(2)	.....	7.45(—)	.....	*Fe* 7.19(5)
3918	8.45(2)	.....	.....	9.14(3)	8.73(3)	8.83(4)	*Fe* 8.65(5); *Fe* 9.07(3); *Fe* 8.43(4); *Fe* 8.33(4)
3920	0.30(3)	0.59(4)	0.50(0)	0.78(4)	0.74(2)	0.84(3)	*Fe* 0.27(10)
3921	1.24(—)	.....	.....	.....	.....	.....	{La^+ 1.56(4)}
3922	2.97(3)	2.48(—)	2.97(0)	3.44(4)	3.38(3)	2.99(3)	*Fe* 2.92(12); S^+ 3.43[6]
3924	.....	4.06(2)	.....	.....	.....	.....	.....
3925	5.61(2)	.....	5.69(?)	5.45(5)	5.25(2)	5.70(2)	*Fe* 5.65(5); *Fe* 5.95(4); *Fe* 5.21(4)
3926	.....	6.22(2)	.....	.....	.....	.....	*Fe* 6.03(3); *Fe* 5.95(4)
3928	8.11(2)	8.15(3)	8.28(?)	.....	8.41(4)	7.97(3)	*Fe* 7.93(8)
3929	9.86(?)	9.50(8)	.....	9.37(4)	.....	.....	{*Fe* 9.22(2); *Fe* 9.12(2)}
3930	0.64(2)	.....	0.54(?)	0.69(8)	0.74(4)	0.65(3)	*Fe* 0.31(8); Eu^+ 0.51[300R]; {Y^+ 0.66(0)}

intensities agree quite well with the laboratory intensities, as is shown in Table IV. There are no strong singlet lines in this region.

Mg^+. The only lines in the region covered are 3848.24 [7] and 3850.40 [6], $3^2D-5^2P^\circ$; as 3848.24 does not appear, the wave-length coincidence of 3850.40 seems due to chance.

Al^+. The strongest line at 3900.66 [10], is blended with *Ti^+* 3900.54 (5). The blend appears in all the stars except θ Leo and α CMa. The triplet 3586.55 [9], 87.06 [8], and 87.44 [7] may possibly be blended with *Fe* 3587.63 (7), which is present in α Cyg.

Si. The only strong line in this region is 3905.53 (8), which possibly blends with the *Fe* line 3906.49 (10). Actually the mean stellar

TABLE IV

	Magnesium Triplet $3^3P^\circ-3^3D$					
	Lab.	⊙	α Lyr	ϵ U Ma	α Gem	α CMa
3838.30...........	100	25	7	7	10	7
3832.31...........	80	15	4	5	7	5
3829.37...........	40	10	3	4	6	3

wave-length is 3905.93; Wright's value in α Cyg is 3906.1. Both measures are definitely smaller than the *Fe* wave-length, indicating the possible influence of the *Si* line. On the other hand, the observed line at 3905.93 is much stronger than 3920.50, which is due to *Fe* 3920.27 of the same intensity (10) in the sun. A weak *Fe^+* line at 3906.04 (−1) may also contribute to the blend and complicate the question.

Si^+. The three strong lines in the region are 3853.67 [3], 3856.03 [8], and 3862.59 [6]. These lines require a considerably lower excitation potential (6.83 volts) than the pair at 4128–4130 (9.80 volts). All three of the lines are present in all the stars except γ Lyr. The intensities usually agree with the laboratory values, as is shown in Table V. There are no other *Si^+* lines in this region.

S^+. There is no evidence for or against the presence of *S^+* in this region, as the strongest line, 3923.43 [6] is blended with *Fe* 3922.92.

Ca. There seem to be no *Ca* lines in this region, as the only coincidence is 3624.63 in α Cyg, intensity 3, which might be due to *Ca*

3624.12 [20]; other members of the same multiplet, however, do not appear.

Ca$^+$. The only lines in the region are 3706.04 [10] and 3736.92 [11]; they are blended with strong *Fe* lines, 3705.58 and 3737.14.

Sc. There are probably no stellar *Sc* lines in this region. A strong line at 3911.83 [100] coincides with an observed line in θ Aur, α CVn,

TABLE V

Si$^+$ LINES

	Lab.	β Ori	α Lyr	γ Gem	α Cyg	α And	θ Aur	α CVn	ϵ UMa	θ Leo	α Gem	α CMa
3862.59..	6	5	7	6	5	4	10	7	2	4	4	3
3856.03..	8	7	7	6	6	5	10	8	4	10	5	5
3853.67..	3	4	3–4	3	4	4	6	6	2	4	3	2

TABLE VI

Lab λ	Lab. Int.	Int. in α Cyg	Blends
3572.48 } 3572.57 }	50	2	*Fe* (5)
3576.33 } 3576.39 }	30	2	*Fe* (4) and *Ni* [3]
3580.93	20	2	*Fe* (30)
3630.74	100	1	*Fe* (3)
3642.83	50	1	....

and ϵ UMa; but the other line of the same intensity at 4023.69 was not observed by Morgan in θ Aur and α CVn.

Sc$^+$. The following lines, which are the strongest in the observed region, may be partly due to *Sc*$^+$ in the spectrum of α Cyg. An examination of the observed intensities shows that *Sc*$^+$ must be a minor contributor. Morgan found rather weak lines of *Sc*$^+$ in the ordinary photographic region of α Cyg (see Table VI).

Ti. Many coincidences in wave-length occur, but they seem due to chance.

Ti$^+$. This spectral region is very suitable for investigations of *Ti*$^+$ because of the presence of very strong lines, such as 3759.30 [200],

3761.32 [200], and 3685.20 [250]. The lines are strongest in α Cyg; they are also very strong in α CVn, θ Leo, and α Gem.

V. The lines shown in Table VII may possibly contribute to blends, but their contribution is still doubtful.

V^+. The strongest lines—3878.75 [35], 3903.26 [25], and 3727.35 [35]—seem present, but they are blended with *Fe* lines, which leaves the identification of V^+ doubtful in this region.

Cr. Many coincidences occur, but they seem due to chance; there are no very strong *Cr* lines in this region.

Cr^+. All lines of intensity ≥ 2 in the solar spectrum are present in α Cyg; even several lines of intensity 1 in the sun may be present,

TABLE VII

Lab. λ	Stellar λ	Mean Observed Intensity	Blends
3855.85 [100]......	56.37	5	Si^+ 6.03; *Fe* 6.38(8)
3850.76 [60].......	40.82	5	*Fe* Mn (10); *Fe* (8)
3828.57 [60].......	28.10	4	*Fe* (8)
3818.24 [60].......	17.70	3	*Fe* (3)
3813.49 [60].......	13.07	4	*Fe* (5); Ti^+(2)
3703.59 [100]......	04.11	H	Hξ

but they are blended with *Fe*. The lines are especially strong in α Cyg and also in α CVn.

Mn. The wave-length coincidences are due to chance; the spectral region does not contain strong *Mn* lines.

Mn^+. The region is also unsuitable for the detection of Mn^+. The peculiar stars, α And and α CVn, which show Mn^+ lines in the ordinary photographic region, contain no trace of 3883.28, which is the strongest line of Mn^+ in this region.

Fe. Very many lines are present; the region is particularly rich in lines of great laboratory intensity. The *Fe* lines are especially strong in θ Leo, α CVn, α CMa, α Lyr, and α Cyg.

Fe^+. Rather few Fe^+ lines are in the region covered; they are strong in α Cyg.

Co. This element seems absent; the strong line 3845.57 [60] coincides with a stellar line; but 3894.10 [60] of the same multiplet is absent; λ 3873.13 [60] is also absent.

Ni. The region is very suitable for the detection of *Ni*. Line 3619.40 [150 R] may contribute to the stellar line 3619.08 in α Cyg, which is mainly due to *Fe* 3618.78; *Ni* 3858.30 [40r] may contribute to 3858.85, which is principally due to *Fe* 9.22. Other coincidences seem due to chance.

Ni^+. The region is particularly suitable for the investigation of Ni^+. The line 3769.46 [5] is strong in α Cyg, intensity 8; it also appears in α CVn and θ Leo. The other strong line is 3576.66 [3], which appears also in α Cyg; it is blended with *Fe* and Sc^+ lines.

Y. The strongest line is 3620.97 [400]; it may contribute to 3621.08 of α Cyg, which is due mainly to *Fe* 3621.47.

TABLE VIII

Lab. λ	Lab. Int.	⊙ Int.
3832.45	100	3N
3788.70	200	2
3774.37	300	3
3709.77	500	3

Y^+. The lines shown in Table VIII contribute to blends.

Zr. The coincidences observed seem due to chance.

Zr^+. The two strongest lines, 3698.17 and 3751.59, are blended with *Ho* and *Hκ*; certain other Zr^+ lines may possibly contribute faintly to blends.

Ba^+. The region is not suitable; it seems improbable that the line at 3891.78 [50] could appreciably blend with *Fe* 91.94.

La^+. The strongest line is located at 3794.77 [500]; it may possibly blend slightly with *Fe* 3795.01. Other coincidences seem accidental.

Ce^+. Higher dispersion is needed; 3801.53 [10] may possibly contribute to 3801.53 in α CVn.

Eu^+. The presence of Eu^+ in α CVn is confirmed.

Rare earths. There are many coincidences with lines of ionized *Pr*, *Nd*, *Sm*, *Gd*, *Tb*, *Dy*, and *Er*, but higher dispersion is required for determining with certainty the reality of the identifications.

PECULIAR A STARS

A number of lines are present in the peculiar stars only; for example, twenty-one lines are present in α CVn but not in the ordinary

Ao stars; there are four similar lines in α And, eight in θ Aur, and nine in ϵ UMa. Many of these lines coincide in position with *Fe* lines, but it is certain that in many cases this identification is not satisfactory.

We wish to express our thanks to Professor Otto Struve, who suggested and encouraged this investigation; to Dr. F. E. Roach, who obtained the spectrograms; and to Dr. W. W. Morgan, who gave valuable advice.

September 11, 1935

THE INTENSITY OF *Hβ* IN THE CHROMOSPHERIC SPECTRUM

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ABSTRACT

Direct photographic measurements made outside of eclipse give a value of 5×10^{10} ergs/cm^2/sec for the energy emitted over a hemisphere in the chromospheric line *Hβ* by a column of gas in the line of sight passing about 2000 km above the photosphere. From this measure the density of hydrogen atoms in the fourth quantum state at the same height is computed as 7 atoms/cm^3, agreeing in order of magnitude with the eclipse results of Pannekoek and Minnaert. The influence of self-absorption is discussed, and it is concluded that observations at 2000 km are not seriously affected by this factor.

The most direct measure of the density of excited atoms in the chromosphere is given by the intensity of the emission lines of the flash spectrum. However, the short time available for observations at the second and third contacts of total eclipses makes photometry difficult, and the first determination of the absolute intensity of lines appears to have been made at the Lapland eclipse of 1927 by Pannekoek and Minnaert,[1] who measured the total energy emitted as *Hγ* radiation by a definite area of the chromosphere, at different heights above the limb. From these data A. Pannekoek,[2] W. H. McCrea,[3] and D. H. Menzel[4] computed the density of hydrogen atoms in the fifth quantum state at the base of the chromosphere as about 135 per cm.^{3} More extensive observations on a number of lines at different heights in the chromosphere were obtained by Menzel at the 1932 eclipse,[5] and further measurements will undoubtedly be made part of the program of future eclipse expeditions.

In view of the importance of such data it seemed worth while to attempt a check measurement outside of eclipse, using the spectroheliograph, with grating inserted, on the 40-inch telescope. The line *Hβ* was chosen for measurement because it can be photographed at moderate heights above the photosphere with relatively little scat-

[1] *Verh. der Amsterdam Akad.*, **13**, No. 5, 102, 1928.

[2] *Ibid.*, **14**, No. 2, 23, 1930.

[3] *M.N.*, **89**, 483, 1929.

[4] *Pub. Lick Obs.*, **17**, 287, 1931.

[5] Unpublished.

tered background light, if the observations are confined to days when the atmosphere is unusually steady.[6]

On July 5 and July 10, 1935, conditions were satisfactory, and eight measurable exposures were obtained on three plates. In addition to the chromospheric spectra each plate contained several exposures of the continuous spectrum at the center of the sun's disk, as well as a set of calibration circles. The disk spectra were taken both before and after the emission spectra and with the same exposure time, 8 seconds. In order to restrict the photometric measures on the plates to small ranges of density, the light from the center of the disk was reduced to an intensity comparable with that of the chromosphere, by two different methods, as follows:

a) An exposed plate of known density was used as a neutral weakener in front of the slit of the spectrograph. Dr. H. Rosenberg very kindly measured the density of this plate with his photoelectric microphotometer, using a blue filter. Similar measures made without a filter showed little difference, and the value of 0.09 was adopted for the fraction of light transmitted under specular illumination.

b) The aperture of the telescope was diminished to 9 inches by means of the iris diaphragm, as compared to 27 inches for the chromospheric spectrum. Both methods were used on each of the plates.

The plate densities were measured on the Ross thermoelectric photometer. All measures were made through a rectangular aperture of sufficient width to cover about 0.5 A of the spectra, which were shifted back and forth so that the readings on the emission lines represented an average across the tops of the lines. Since $H\beta$ is relatively flat-topped at the heights above the chromosphere at which the observations were made, and has only weak wings, the total emission can be taken as that of a rectangular line of the same intensity and of a width equal to that of the observed line at half its maximum intensity. Comparison was made with the continuous spectrum between the absorption lines, for which Abbott's data for the years 1920–22 were used, corrected for the absorption of the lines by means of the table given by G. F. W. Mulders.[7] The value

[6] Cf. *Ap. J.*, **75**, 282, Pl. XIII, 1932.

[7] Utrecht dissertation, Table XVI, 1934.

adopted for πI, the energy emitted over a hemisphere by 1 cm² of the sun's surface per second per angstrom at $H\beta$, was 1.32×10^7.

The heights above the photosphere of the points measured were estimated roughly by comparison with plates taken for a previous investigation, to which reference is made for a discussion of the method.[8] In forming Table I the eight points measured were combined into three groups according to altitude, because of the uncertainty in the individual heights.

Successive columns give the mean height, the half-value width, the ratio of intensities of the chromospheric line to the disk spectrum as

TABLE I

INTENSITIES OF $H\beta$

Mean Height	Line Width	R_{Dia}	R_{Screen}	R_{Mean}	πI	N_4
1500 km	1.0 A	0.039	0.032	0.036	5.2×10^5	6.1×10^{10}
2100	1.0	.036	.029	.032	4.6	5.4
3100	0.9	0.023	0.022	0.022	2.8	3.3

photographed with the iris diaphragm, the same ratio when the absorption screen was used, the mean of the two ratios, the radiation of the chromospheric line in ergs per square centimeter per second, and the total number of hydrogen atoms in the fourth quantum level in a column of 1 cm² cross section passing through the chromosphere at the height of the observations. If we neglect self-absorption, this last quantity is given by the expression:

$$\tfrac{1}{4}N_4 A_{42} h\nu = \pi I \; . \qquad (1)$$

Since the values in the last two columns decrease more slowly with height than any previously determined density gradient would predict, it is probable that atmospheric scintillation produced a smoothing effect by superimposing the light from adjacent levels. For this reason we do not attempt to derive a density gradient from our data, but conclude only that in the middle chromosphere (near 2000 km) N^4 is approximately 5×10^{10}.

[8] *Ap. J.*, **75**, 284, 1932.

Adapting to $H\beta$ the equations summarized by Menzel,[9] the density of excited atoms at the level of observation is given by

$$(n_4)_{2000} = \sqrt{\frac{a}{2\pi R}} \cdot N_4 \,. \tag{2}$$

If for a, the exponential constant defining the density gradient, we substitute 0.8×10^{-8}, which is a rough mean of the values found for different years by Menzel, we obtain

$$(n_4)_{2000} \sim 7 \text{ atoms/cm}^3 \,.$$

We can only guess at the distribution of the hydrogen atoms among the different levels; but if an effective temperature of 5800°, as adopted by the earlier investigators, is assumed in computing the Boltzmann factors, we obtain

$$(n_1)_{2000} \sim 10^{11} \,, \qquad (n_5)_{2000} \sim 6 \,, \qquad (n_2)_{2000} \sim 280 \,,$$

corresponding to a pressure of about 10^{-7} atmospheres for hydrogen.

Pannekoek's density and gradient lead to a density of 5.5 for $(n_5)_{2000}$, which agrees well with the value obtained here; but it must be noted that the higher gradient found by him leads to densities at the base of the chromosphere which exceed ours by a factor of 2, if, on the basis of Menzel's gradients, we use the relation

$$(n_r)_0 = (n_r)_H e^{-0.8 \times 10^{-8} H} \,, \tag{3}$$

where H is the height expressed in centimeters. We find

$$(N_1)_0 \sim 10^{12} \,, \qquad (n_5)_0 \sim 90 \,, \qquad (n_4)_0 \sim 105 \,, \qquad (n_2)_0 \sim 4 \times 10^3 \,.$$

These densities of excited atoms are considerably lower than those obtained in Menzel's recent work, but the higher temperatures for which he finds evidence would reduce the Boltzmann factor so that the total hydrogen pressure at the base of the chromosphere may well be less than the value of 10^{-6} atmospheres indicated here. At the

[9] *Pub. Lick Obs.*, **17**, pp. 242 and 288.

present time the accuracy of our knowledge of chromospheric densities is limited chiefly by our uncertainty as to the effective temperature of excitation.

The effect on the measured intensities of the self-absorption of the chromospheric gases must be considered. If in any column of gas we let I_ν represent the total intensity of radiation of frequency ν which would be emitted if there were no self-absorption, and I'_ν be the observed intensity, then it follows directly from the equation of transfer that

$$\frac{I'_\nu}{I_\nu}=\frac{1-e^{-\tau_\nu}}{\tau_\nu}, \tag{4}$$

where $\tau_\nu=\alpha_\nu N_r=\alpha_\nu \dfrac{n_r}{n_s} N_s$, if the emission line represents a transition from state s to state r. This expression was derived by Menzel[10] and is equivalent to the formula set up somewhat less generally by Pannekoek.[11]

If the line has a Doppler profile, we can write

$$\alpha_\nu=\alpha_0 e^{-\omega^2} \tag{5}$$

where, in the notation of Mitchell and Zemansky,[12]

$$\omega=\frac{2\sqrt{ln 2}\cdot\nu}{\Delta\nu_0} \tag{6}$$

$$\alpha_0=\frac{k_0}{n_2}=\frac{2}{\Delta\nu_D}\sqrt{\frac{ln 2}{\pi}}\cdot\frac{\lambda_0^2}{8\pi}\cdot\frac{g_s}{g_r}\cdot A_{sr} \tag{7}$$

and the Doppler breadth,

$$\Delta\nu_D=2\sqrt{ln 2}\cdot\frac{\nu_0}{c}\cdot u, \tag{8}$$

if u is the mean random velocity of the atoms.

[10] *Ibid.*, eqs. (10. 13) and (10. 15), p. 241.

[11] *B.A.N.*, 4, 263, 1928.

[12] *Resonance Radiation and Excited Atoms*, pp. 94–104, 1934.

By integrating with respect to ω, we obtain for the ratio of total intensities

$$\frac{I'}{I}=\frac{1}{\tau_0\sqrt{\pi}}\int_{-\infty}^{\infty}(1-e^{-\tau_0 e^{-\omega^2}})d\omega\ , \tag{9}$$

which is the function S computed by Ladenburg and Levy[13] and given in part by Mitchell and Zemansky in Appendix V of *Resonance Radiation*. Then

$$\frac{I'}{I}=S=\frac{\tau_0 S}{\tau_0}=\frac{N_2 S}{N_2}=\frac{N_4 S}{N_4}\ . \tag{10}$$

Since we observe I' we must use, instead of equation (1), the expression

$$\tfrac{1}{4}A_{42}h\nu\cdot N_4 S=\pi I'\ , \tag{11}$$

and thus determine actually $N_4 S$, whence we obtain $N_2 S$ and $\tau_0 S$. Since $1/S$ is a continuously increasing function of τ_0 and of $\tau_0 S$, there is a definite value of $\frac{I}{I'}$ corresponding to each value of $\tau_0 S$. The relationship is shown in the curve of Figure 1, computed from the tables cited above.

We can now determine the true intensity corresponding to the observed emission of $H\beta$ at 2000 km. Since $n_2 S$ was found to be 280, $N_2 S$ is 2.1×10^{12}. If the thermal motion of 10 km/sec is taken as the random velocity of the hydrogen atoms, a_0 becomes 8.7×10^{-14} and $\tau_0 S$ is 0.18. From the graph the ratio I/I' is seen to be 1.05; so the neglect of absorption in the reduction of these observations appears to be justified. If the Boltzmann factor relating n_2 to n_4 has been taken as too small by a factor of 5 or more, this conclusion is vitiated; but since, as was remarked above, the actual effective temperature is probably higher than 6000°, the computed absorption is more likely to be a maximum than a minimum value, the more so since the introduction of an additional velocity of turbulence would diminish a_0. However, it must be admitted that the considerable excess in the observed width of the line over that predicted by the Doppler

[13] *Zs. f. Phys.*, **65**, 189, 1930.

formula would suggest the presence of absorption, although there may be some other explanation of the fact. It should also be noted that the data here indicate strong self-absorption for $H\beta$ and $H\alpha$ at very low levels in the chromosphere, where the opacity is ten or twenty times greater.

The curve of Figure 1 could be extended to greater opacities by substituting the natural damping profile in place of the Doppler

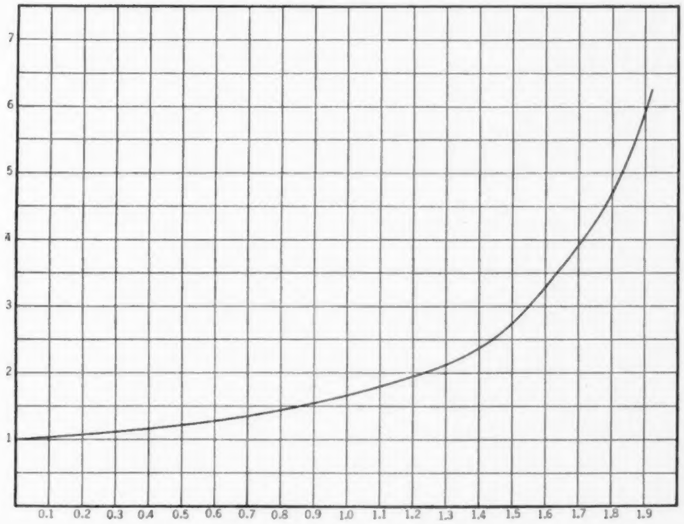


FIG. 1.—$H\beta$ in chromospheric spectrum

form, but this has not been done because it appears that equation (4) is inadequate for higher densities, where, as Menzel pointed out,[14] it fails to predict the observed double reversal of the strong lines. By the introduction of an excitation gradient in the sense that the effective temperature gradually decreases toward higher levels, it is possible to account for the presence of central absorption lines and to show that their appearance will not at first greatly affect the total absorption; but the computation of the profiles involves so many assumptions that it seems scarcely worth while to pursue the subject farther until the shapes of the observed lines are measured

[14] *Pub. Lick Obs.*, **17**, p. 241.

with greater precision. The formula derived by Unsöld was based upon assumptions of a particular method of excitation and of a density distribution, which no longer appear tenable; and the densities estimated here on the basis of it[15] must be discarded.

I am indebted to Dr. Menzel for the opportunity to make comparisons with his recent results and to refer to them in advance of publication.

YERKES OBSERVATORY
September 1935

[15] *Ap. J.*, **75**, 296, 1932.

OBSERVATIONS OF RADIAL MOTIONS OF PROMINENCES

PHILIP C. KEENAN

ABSTRACT

Maximum radial velocities of prominences observed between 1930 and 1935 show no marked preference for either approach or recession. The majority lie between 30 and 70 km/sec, with an average of 53 km/sec. High positive and negative speeds frequently appear nearly simultaneously in adjacent parts of the same prominence.

A tentative division into three types is suggested for prominences which develop large radial motions.

By the use of a motion-picture camera on the spectrohelioscope it is possible to record velocity shifts. Such films can provide all the data necessary for the construction of the true three-dimensional trajectories of knots of gas exhibiting definite mass motions.

The spectrohelioscope of the Yerkes Observatory was set up and adjusted by W. W. Morgan and F. E. Roach in 1929, and since 1930 it has been used to supplement the work of the spectroheliograph on the 40-inch telescope, particularly through the use of the line-shifter, to measure radial velocities of prominences. Since limitations of space made it necessary to obtain the 2-inch image of the sun by means of a telephoto lens system, designed by F. E. Ross and Y. C. Chang, there is considerable loss of light at the component lenses, and it is necessary to work with wider slits—usually 0.14 mm and 0.11 mm—than have to be employed with a single objective. With these openings it is not usually feasible to measure velocities of less than 15 km/sec, but shifts corresponding to velocities greater than 20 km/sec can be determined with an uncertainty of about 5 km/sec.

The observations have been directed mainly to the measurement of the greatest positive and negative radial velocities present in prominences showing distinct activity, the maximum being defined by the largest setting of the line-shifter at which any fairly conspicuous portion of the prominence attains its greatest brightness.

The 102 maximum velocities measured in the interval from 1930 to 1935 are a fair sample of the active prominences during the last sun-spot minimum, although the observations were somewhat sporadic during the first two years. Table I lists the number of velocities within each interval of 5 km/sec but is of significance only for speeds

exceeding 20 km/sec. From the data it is evident that large velocities of approach and of recession are equally probable. The frequency curve has a rather flat maximum extending over speeds of 30–70 km/sec, and very few prominences possess horizontal mass velocities exceeding 100 km/sec. The maximum observed was −108 km/sec, on June 30, 1933. Near the phase of spot maximum it may be expected that high speeds will be more common.

TABLE I

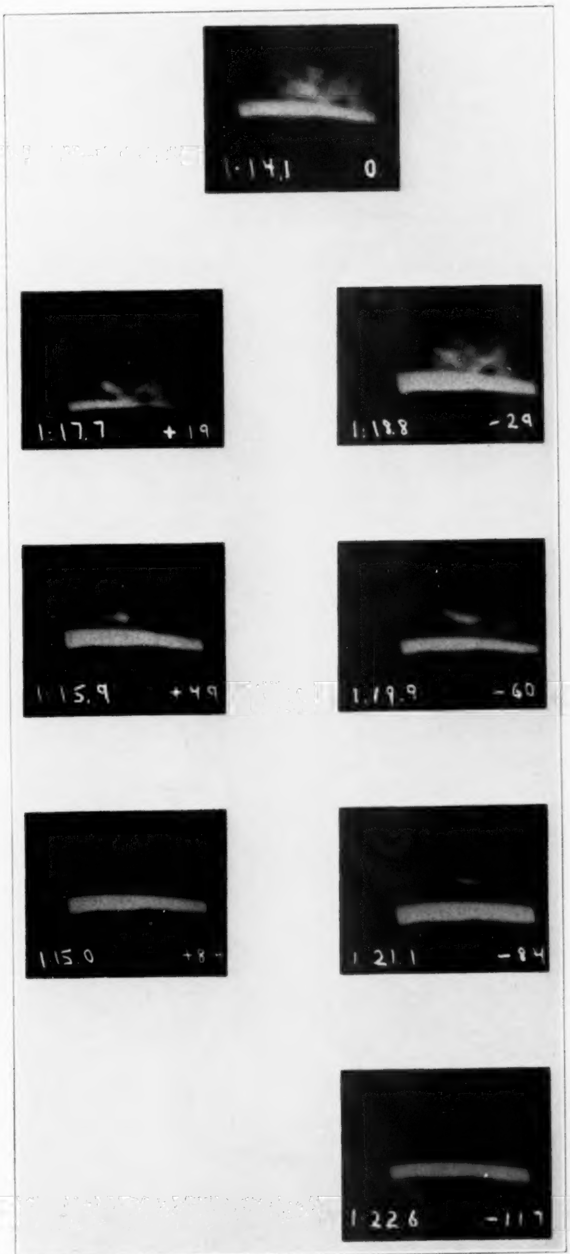
MAXIMUM RADIAL VELOCITIES

Range (Km/Sec)	No.	Range (Km/Sec)	No.
> −100	1	> +100	....
−100 to −90	5	+100 to +90	1
−90 to −80	2	+90 to +80	3
−80 to −70	2	+80 to +70	1
−70 to −60	8	+70 to +60	14
−60 to −50	6	+60 to +50	6
−50 to −40	12	+50 to +40	16
−40 to −30	8	+40 to +30	7
−30 to −20	4	+30 to +20	4
−20 to −10	1	+20 to +10	....
−10 to 0	1	+10 to 0	....
Total	50	Total	52
Average velocity	−54.2 km/sec	Average velocity	+52.5 km/sec

A striking characteristic of almost exactly half of the active prominences observed was the association of high positive and negative velocities in the same prominence. Very often the extreme speeds in opposite directions occur nearly simultaneously in adjacent portions of a prominence, and probably indicate the explosive repulsion of discrete streamers and masses of gas from a common center (see Plate IV). Shortly after such outbursts occur, the moving gases are sometimes seen as brilliant, nearly horizontal streaks which shift vertically by a small amount but show no marked displacement along the limb as the line-shifter is rotated. Here the material must be spreading outward in waves or sheets which later break up into smaller masses.

The prominences which show large radial motions may be divided tentatively into three groups, which, though often seeming to merge

PLATE IV



RADIAL MOTIONS IN A PROMINENCE
(August 26, 1935)

into one another, probably represent real differences in the nature of the prominence involved and in the forces giving rise to the motions. These are:

a) Spot-type or fountain prominences. These are usually small jets or arches which change with great rapidity; the total time between the emergence of a jet and its subsidence or fading-out is often of the order of 10 minutes.

b) Portions of active prominences, which suddenly brighten and develop high speeds, lasting anywhere from a few minutes to several hours. The outbursts described in the preceding paragraph are usually of this type.

c) Streamers to centers of attraction. In some cases these have been observed to endure for 5 hours or more, with almost constant velocity shifts, while in others there is an intermittent ejection of knots of matter which seem to follow one another along nearly the same path. While some of the fields of force causing these streamers are ephemeral, the group as a whole includes nearly all cases of persistent motion.

It is of interest to compare the prominence data with the excellent measures of the velocities of dark $H\alpha$ flocculi made by H. W. Newton.[1] He found that the frequency distribution of flocculi associated with sun-spots have a pronounced peak between 20 and 40 km/sec, while those that did not accompany spots had only very small displacements. Although no systematic observations of flocculi have been carried out here, we have noted 21 markings ascending at an average speed of 52 km/sec and 29 descending at an average speed of 45 km/sec, the higher mean rates being due to the neglect of small shifts in our records. The tendency of dark flocculi to flow downward into or near sun-spots, of which Hale had observed several instances,[2] was shown by Newton to be a common characteristic and has been noted repeatedly here. However, it is not unusual for an arched eruption to cross over a spot, descending on the side opposite that on which it rose; and in some groups a flocculus dropping into one spot will apparently be issuing from another at its far

[1] *M.N.*, **94**, 472, 1934.

[2] *Ap. J.*, **71**, 73, 1930.

end. All of these phenomena are consistent with the behavior of calcium prominences around centers of attraction near the limb.[3]

The usefulness of the spectrohelioscope can be greatly enhanced if the record is made photographic. Following the successful pioneer work of Petrie and McMath with their spectroheliokinematograph,[4] experiments were begun here in the summer of 1934 with the help of P. Eberhart and E. L. McCarthy, and it was found that promising results could be obtained by placing an ordinary motion-picture camera behind the second slit and rotating prism. With Eastman Supersensitive Panchromatic film, prominences can be photographed on a scale of 62,000 km to the millimeter with exposures of 10–20 seconds. Before good pictures could be obtained, it was necessary to replace the old spring drive of the coelostat with a Telechron motor and new gears, which were designed and assembled by C. Ridell with the co-operation of Dr. G. W. Moffitt.

An example of the use of the instrument to make a rapid record of velocity shifts is shown in Plate IV, in which the top exposure was taken at the center of the line and positive and negative settings increase downward in the right- and left-hand columns, respectively. Below each image the Central Standard Time is given in the left corner, while the number on the right is the velocity shift, in kilometers per second, defined by the center of the slit. There was no great change in the appearance of the prominence during the time required to obtain this set of 15-second exposures.

Photography is advantageous in the case of complex prominences in which the components change their shapes and speeds so rapidly that they cannot be drawn accurately from visual observations. However, the greatest importance of the method lies in the possibility of using a single set of exposures to carry out a three-dimensional analysis of the path followed by any part of a prominence which shows a definite mass motion.

As an example we take a knot of matter which emerged from a low active prominence on the west limb on the morning of September 9, 1935, and described a flat arch, entering the disk again some 80,000 km south of its starting-point 27 minutes after it was first seen. Dur-

[3] *Ap. J.* (in press).

[4] *Pub. Obs. Univ. of Michigan*, **5**, 103, 1934.

ing the interval thirteen exposures were obtained with the camera as the spectrohelioscope.

With the tangent to the sun's limb chosen as the x-axis, the vertical to the solar disk at the limb as the y-axis, and the line of sight

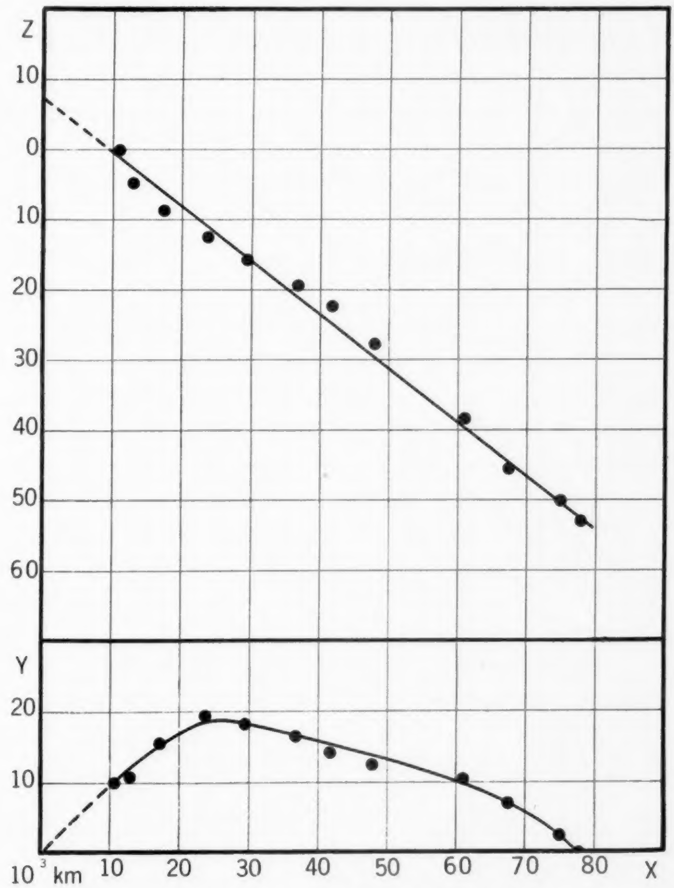


FIG. 1.—Motions in prominence of September 9, 1935

from the earth as the z-axis, the x and y co-ordinates of the tip of the moving knot were measured directly on the film as a function of the time. In Figure 1 the apparent course followed by the knot is plotted as the lower curve, which represents the projection of the space trajectory on the xy-plane. Obtaining v_x and v_y from the diagram and knowing v_z, the measured radial velocity for a given point

on the apparent curve, it is easy to determine graphically the approximate displacements in the z-direction between the recorded times of exposure. In this way the projection of the motion on the xz-plane was found and is plotted in the upper part of the figure. Within the large limits of error due to the small scale of the image and the integration of velocities, the path of the knot, as seen from above, can be represented as a straight line, which means that the plane of the arch was perpendicular to the sun's surface. It will be of considerable interest to determine how often this holds true, for there is reason to believe that the arches sometimes make large angles with the vertical, and in at least one case the radial velocities give definite evidence of a spiral motion.

This prominence is offered merely as an illustration of the method of attack. Although small instruments can do useful work, accurate determination of space motions will require equipment powerful enough to permit shorter exposures and greater resolving power. Such an instrument will then be of the greatest value in studying the forces affecting prominences, particularly when used in conjunction with an ordinary spectroheliograph, which is superior in the delineation of fine detail but gives only a two-dimensional picture and may fail entirely to record prominences moving with large radial velocities.

I should like to express my appreciation for constructive suggestions received from the members of the Yerkes Observatory staff, from Dr. Bartky of the Astronomy Department of the University of Chicago, and from Dr. Pettit of the Mount Wilson Observatory.

YERKES OBSERVATORY
September 1935

REVIEWS

Relativity, Gravitation and World-Structure. By E. A. MILNE. Oxford, 1935. Pp. viii+365; Figs. 21; Pls. IV. $8.00.

In this volume Milne presents, under a unified point of view, the results of his investigations during the last three years on the structure of the system of the extra-galactic nebulae. He constructs an idealized model of the universe from foundations differing in certain respects from those of current acceptance; in terms of it he offers explanations of such apparently varied phenomena as red-shift in light from extra-galactic nebulae, K-effect in the remoter stars of our own galaxy, interlopers from without, cosmic rays, and the clouds of calcium and sodium vapor accompanying the galaxy. I attempt in this review to indicate briefly the main points leading to these proposed explanations and to evaluate them.

Milne begins by arguing the relative arbitrariness of kinematics and physical law, and announces his intention to pursue the kinematical aspect alone. This he proposes to do in terms of a strictly operational methodology, guided by an a priori uniformity postulate which he fittingly calls "the cosmological principle." No theory of gravitation is posited; indeed, he believes himself able to derive, by kinematical and statistical considerations alone, a gravitational law sufficient for his immediate purposes (p. 181), and is thus led to "the inspiring possibility that all 'gravitational' situations ought to be capable of similar treatment" (p. 274). The cosmological principle demands that in the smoothed-out idealization all observers of a given class ("fundamental particle-observers") are completely equivalent, in the sense that their views of the life-history of the universe are identical—as such it is, despite Milne's protestations to the contrary, equivalent for the case in point to the uniformity postulate on which the general relativistic treatment of the problem is based. The experiences of such observers are analyzed in chapter ii, allowing them the use only of clocks, theodolites, and light-signals. A promising attack on the problem of collinear observers suffering relative acceleration is initiated, but an attempt to extend the results there obtained to the full three-dimensional problem fails. Milne accordingly obtains only a solution of the problem for relatively unaccelerated observers, whose mutual relations are given by Lorentz transformations. This system of fundamental particle-observers, later to be identified with nebular nuclei (p.

123), may be mapped in a Minkowski space-time in which the world-lines of the observers consist of all rays originating in a given event O ("creation") and extending into the future. As I hope to show elsewhere in this *Journal*, Milne's failure to obtain the solution of the problem in which the observers are relatively accelerated is attributable to his imposition of restrictions foreign to the general nature of the program, and that on avoiding them one is led directly to the invariant theory of that general line element on which the general relativistic theory of cosmology is based—although the gravitational theory implied by the latter need by no means be accepted.

In chapter v the acceleration of a test particle is analyzed, subject to the conditions imposed by the cosmological principle, and is found to be determinate to within a single dimensionless function G of two (single-valued) scalars X and ξ, the former of which has the dimension $(\text{time})^2$ and the latter of which is dimensionless. Milne then maintains that G can depend at most on the dimensionless variable ξ, as no fundamental constant of purely temporal dimensions is available to enable the formation of a dimensionless function involving X. This is a point of great significance for his further developments; at best it is valid only if he is ultimately successful in building up a complete gravitational theory from kinematics and statistics alone, and is certainly untenable in dealing with the problem of accelerated nebulae, for there one can obtain from the given motions a dimensionless function of X (e.g., from the Doppler shift-ratio discussed on p. 35). In any case I fail to understand the import of his partial concession:

> If universal constants exist, the function G controlling the behaviour of *additional* test-particles may be of the form $G(X, \xi)$ instead of simply $G(\xi)$. But the motion of the *given* particles in the simple kinematic system is unaffected [p. 112 n.].

In Part III Milne supplements the fundamental system with a statistical ensemble of further particles in such a way that the cosmological principle remains satisfied for the fundamental observers; its proper particle density in phase space is found to be of the form $\psi(X, \xi)/c^6X^{\frac{3}{2}}$, and again Milne argues that the dimensionless function ψ cannot depend on X. The condition of permanence of such a system, which has here the character of a physical law of conservation, is shown to require that the function G determining the accelerations be of the form (eq. [21], p. 180)

$$G(\xi) = -1 - \frac{C}{(\xi-1)^{\frac{3}{2}}\psi(\xi)}$$

where C is an undetermined dimensionless constant "thrown up by the integration"—all on the assumption that G and ψ may not depend on X. Herein lies the heart of his theory of gravitation, for on the next page he concludes that this formula

> determines the acceleration of every particle present. For it fixes $G(\xi)$ in terms of $\psi(\xi)$ and a single undetermined constant C, ... Nothing further can be demanded from any "theory of gravitation" than that it prescribes the acceleration of every particle present in the system, and also the acceleration of any free particle added to the system. Relation (21) achieves both.

But on the same page he admits: "We have not obtained a general theory of gravitation." This being the case, it would seem premature to exclude the possibility that a less idealized description of the physical universe might demand the introduction of a theory of gravitation containing a universal constant with the aid of which a fundamental constant of purely temporal dimensions could here be constructed—as, e.g., the combination $a=Km/c^3$, where K is the Newtonian constant, c the velocity of light, and m a mass associated with the given particle-observers. G or ψ could then depend on X as well, and the argument leading to his relation (21) between them breaks down; for the case in which ψ is still independent of X one obtains a relation of the same form, but in which C may be an arbitrary dimensionless function of X. This eventuality casts grave doubts on the validity of Milne's claim (in continuance of my last quotation):

> But we have obtained the complete gravitational behaviour of a certain family of systems, namely those described by any arbitrary function $\psi(\xi)$, and that without recourse to any assumptions about the nature of gravitation, or the introduction of any empirical constants.

And certainly the corresponding argument cannot hold in any case in which the fundamental observers themselves are in accelerated motion.

The integration of the equations of motion for $G=G(\xi)$ is carried out in chapters viii and xiii of this part, and leads to most fearful and wonderful results in the sequel. In essence, he demands that the world-line of each particle of the statistical system be contained completely within the light-cone with vertex at the singular event O, that it be tangent at O to the world-line of some one of the fundamental observers (say the one characterized by the vector velocity V_0), and that it approaches parallelism with the world-line of some other fundamental observer V_0' as $t\to\infty$. He rightly finds that in order for this behavior to be possible $G(\xi)$, and therefore the acceleration, must be a double-valued function of the variable ξ; which of the two branches is to be chosen at a given event in the trajectory

of a particle depends on its previous history. The way to this conclusion has been prepared by an argument (p. 150), which I am unable to follow, tending to show that ξ must increase monotonically from its value 1 at O to the value ∞ at some finite time t_l in the career of the particle—at which point the particle acquires the velocity of light! Since $(\xi-1)^{\frac{1}{2}}\psi(\xi)$ is shown to have a zero at $\xi=\infty$ (p. 235), Milne chooses the other branch of this double-valued function for the determination of the rest of the trajectory, on which ξ decreases monotonically toward 1 as $t\rightarrow\infty$; this is equivalent to changing the sign of the constant C in the formula given above for G. Each particle of the statistical system thus passes through the five stages: (i) it originated at O with a certain velocity $\boldsymbol{V}_0$, (ii) it is accelerated radially away from that fundamental observer who is characterized by velocity $\boldsymbol{V}_0$ until (iii) at a finite time t_l it acquires the velocity of light and (iv) is subsequently decelerated until (v) its motion parallels that of a certain fundamental observer of velocity $\boldsymbol{V}_0'$.

Milne now proposes (p. 196) as his idealized model of the universe one in which there is associated with each nebular nucleus (i.e., fundamental particle-observer) a three-parameter family of particles having the same initial velocity $\boldsymbol{V}_0$; the resulting six-parameter family of additional particles constitutes the statistical system. Each pair of nuclei moves apart with a relative velocity which is directly proportional to their separation at any given time of observation, thus accounting for the observed velocity-distance relationship in extra-galactic nebulae. The members of each subsystem "are distributed in density around their nucleus according to a law of the inverse cube, or a law somewhat more severe" (p. 277), and recede individually as described above. While they are still more or less associated with their parent-nucleus, they will be observed to show a K-effect in radial velocity; Milne remarks (p. 198): "This is a well-known feature of the remoter [B-type] stars of our own system, and it here finds its explanation." On escaping from the nucleus they may appear as "interlopers" moving through another nebula; evidence for this behavior is found (p. 199) in Larmor's observations on "moving clusters which, though now inside our own system, appear to have entered from the outside." During the stage in which their velocities are near that of light they will be observed as cosmic rays (chap. xii)! And when they subsequently slow down and become associated with another nucleus of velocity $\boldsymbol{V}_0'$ they will give rise to "cosmic clouds," such as those accompanying our own galaxy (chap. xiii). A very readable summary of these and other predictions, representing the principal results of the book, is given in chapter xiv, "The Career of the Universe."

In the final part, IV, Milne discusses the relationship between his kinematical solution and that offered by the general relativistic theory of cosmology. The latter is emphatically rejected in favor of the former; the argument in which he seems to have greatest confidence is that, from the strictly operational standpoint, the "mass-conserving" relativistic models allow the apparent "creation" of matter, i.e., its sudden appearance at the boundary of what is in the relativistic literature called the "observable universe." It is difficult to see in what sense "this creation of matter within the temporal experience of the observer is in contradiction with physics" (p. 335). Milne asserts (p. 341), also without further justification: "The features of our statistical kinematic model . . . . are beyond the present resources of 'general' relativity"—although in so far as his speculation on cosmic rays and the dependence of acceleration on previous history are concerned this may of course be granted without further elucidation.

The descriptive parts of the book are, on the whole, interestingly written—enlivened in spots by such bursts of enthusiasm as in his remotely nostalgic panegyric on "the confines of the universe" (pp. 137–38), by such trenchant sincerity as his description of the oscillating universes (p. 337) as "the fantastic weavings of the mathematical loom, orgies of mathematical licence," and by such beautiful thoughts as (p. 273): "General relativity is like a garden where flowers and weeds grow together. . . . . In our garden we try to cultivate only flowers" (!). In the first part of the book Milne sets up a relativistic strawman (e.g., p. 119), gleaned largely, I presume, from analogues found in the semipopular expositions of Eddington and Jeans, and then turns upon it with the asseveration: "In this book I am more concerned with constructive results than with tilting at obsolescent modes of thought" (p. 120). But undoubtedly the greatest drawback in the exposition, for the mathematically expert and inexpert alike, will be found in the cumbrousness and obscureness of the mathematical parts. Instead of applying fundamental results long established by invariant and group theory, Milne plows through pages of complicated analyses, involving mainly functional and differential equations, which are in effect but roundabout proofs, for the specific cases in point, of these well-known results. The adoption—and, if deemed necessary, a preliminary exposé—of these more enlightened methods would enable him to reduce whole sections to relatively easy and lucid analysis.

Clearly a book with such broad claims must arrest the attention of all scientists engaged in the fields with which it deals, for it issues a challenge which cannot be ignored. To what extent, then, must a strictly opera-

tional attack on the problem, with the aid of the "cosmological principle," necessarily lead to Milne's conclusions? In so far as the kinematical relations between the fundamental observers is concerned, Milne's solution is but one of the host of possibilities corresponding to the manifold of all types of relative nebular motions—as determinable by Doppler effects—and is but one of the three or more in which the nebulae are relatively unaccelerated. In any type involving acceleration there seems no hope of establishing a gravitational theory along the lines laid down by Milne, for here there exists a dimensionless function of X which may enter into the dimensionless function G determining the acceleration of the auxiliary particles, and the Boltzmann condition of permanence can no longer require an essentially unique relation between G and the distribution function ψ; in my opinion there exists almost as little hope of its justification in the cases in which the fundamental observers are unaccelerated. This is by no means an unmitigated woe, for the extra latitude allowed should make possible the construction of cases reproducing all essential properties of Milne's system without the introduction of double-valued accelerations —as, e.g., that simple adaptation $G=-a/X^{\frac{1}{2}}$ of the Newtonian gravitational theory to Milne's own kinematics, which reproduces all the above-described features of the motion with the exception of that rather alarming one concerning the origin of cosmic rays. In conclusion, then, the whole book should be approached in that spirit advised by Milne in discussing the apparent ambiguity of acceleration (p. 246)—"The reader must not allow himself to be humbugged by mathematics."

H. P. ROBERTSON

ERRATUM

Dr. A. Unsöld has kindly called my attention to the fact that in *Ap. J.*, **79**, 431, 1934, the coefficient 2 should be dropped in formula (5). The actual computations were made with the correct formula and were checked graphically.

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